Throughput Performance of Network-Coded Multicast in an Intermittently-Connected Network

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***Abstract*—Consider an intermittently-connected mobile network consisting of n relay nodes, a single source node, and m destination nodes exhibiting a stochastic model for mobility. Each mobile relay node is also equipped with finite storage. We seek to analyze the performance of Multicast enabled by Network Coding in such a network under the store, carry, and forward paradigm, and compare its performance to a simple custodial-multicast scheme. Though accurate analysis of network-coded multicast is very complicated, we derive a provable way to obtain tight bounds on the performance. We then develop a queuing-theoretic framework to analyze the steady-state throughput performance of the network-coded scheme under this setup, which is then solved iteratively. The framework developed thus enables speedy evaluation of the communication protocols described. Our analytical results, supported by simulation studies, show that the network-coding-based scheme offers considerable improvement for the case when the storage size of the relay nodes is small and when the number of destination nodes is large.**

I. Introduction

Intermittently-Connected Mobile Networks (henceforth referred to as ICMANETs) constitute a new class of ad-hoc networking architecture that has drawn much attention in the field of wireless networks recently. Often, communication devices are deployed with very little backbone support and exchange information in a collaborative fashion. Such infrastructure-less networks often occur in applications such as wildlife and habitat management [1], defense networks, vehicular networks, and in networks providing cheap, basic internet connectivity to rural areas in developing nations. Depending upon the application context, they may also be known as Delay Tolerant Networks (DTNs). Typically, ICMANETs are characterized by the lack of end-to-end communication paths, lack of acknowledgement messages, opportunistic communication over intermittent links, and hence large communication delays.

Conventional Mobile Ad-hoc Networks (MANETs) rely on the existence of end-to-end paths between the source and destination nodes in spite of node mobility. However in ICMANETs, multi-hop paths through which information is sent evolve in space and time. Such special constraints posed by the latter make traditional communication protocols inefficient, and new ones are required in their place. For example, while certain routing schemes designed for general ad-hoc networks such as Dynamic Source Routing [2] fail in such networks, several efficient schemes tailored to these networks have been devised in the past [3]–[5]. These schemes use the “store, carry, and forward” paradigm (also known as “mobility-assisted routing”) for message delivery, wherein the source node opportunistically transmits packets (intended for a specific destination) to any other node that it comes in contact with, and relies on the mobility of these “relay” nodes to transmit them to the intended destination. In these schemes, end-to-end paths are created by node mobility in a continuous space-time evolution of the connectivity graph.

In order to enhance the information-transmission capacity of ICMANETs limited by the lack of connectivity, network-coding techniques such as random linear coding (RLC) have been proposed [6]. The motivation behind these schemes comes from the *coupon collector effect* [6] by which the average latency of information transmission can be considerably reduced. In this work, we seek to quantify the benefits of such a network-coded scheme for multicast communication under an analytic ICMANET model. In particular, our quantity of interest is the average steady-state rate at which packets can be sent from a single source to m different destination node, by employing the aforementioned store, carry, and forward paradigm in a network of n mobile relay nodes. We seek to understand the effects of various parameters such as buffer-size at relay nodes, communication range, and phenomena such as interference. In addition, we are interested in obtaining regimes under which network coding performs significantly better than a simple custodial scheme which implicitly replicates packets from the source m times (i.e., as many times as the number of destinations served). We find that network coding has the dual benefit as a result of better buffer management and as a result of coupon-collector effect. However, we observe that significant improvements are achievable only when the number of destinations served (m) is large and when the relay-node buffer sizes are not very large.

The problem of accurately modeling the performance of ICMANETs is of natural interest since the performance models of conventional MANETs are not applicable due to the aforementioned reasons. Further, simulation techniques to obtain

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the steady-state performance of such a network under network network coding is infeasible due to (1) the rather large settling time involved and (2) the complexity of continuous decoding of the packets at the destination. In this paper, we seek to show that in such cases wherein exact analysis or simulation is infeasible, it is possible to obtain rigorous upper and lower bounds by iteratively solving for the steady-state distribution of a low-complexity Markov Chain. We demonstrate this for the communication scenario we address in this paper, viz., network-coded multicast. Also, we show that that this task is possible while taking into consideration practical constraints such as limited node storage, random contact times, and contention between nodes. We also show that the proposed framework is applicable the general class of mobility models that exhibit stochastic stationarity. We achieve this task using queuing theory and Embedded-Markov-chain techniques. Finally, we validate the analysis using simulations for the random-waypoint mobility model.

II. Related Work

Performance modeling of mobile ad-hoc wireless networks, in particular, delay and throughput effects, has attracted considerable attention in the recent past [7]–[9]. Pioneering work in the study of the capacity of mobile ad-hoc wireless networks was due to [7]. Due to the differences in the communication paradigm mentioned in the previous section, we expect significant differences in the performance characteristics of ICMANETs compared to those of general MANETs which rely upon end-to-end connectivity. Additionally, asymptotic results almost never hold for ICMANETs since they are typically characterized by sparseness.

In the context of intermittently-connected networks, the lack of an end-to-end path from a given source to the destination has been considered in similar DTN-related work such as disconnected mobile networks [1], [10], [11] and other forms of DTNs [3], [12]. Mathematical modeling of the performance of ICMANETs has drawn considerable attention recently. Though performance modeling of DTNs and other cases of ICMANETs has been visited in the past, they very often tend to use Poisson-process-based models for contact times [13], [14]. We have shown in the past how such an assumption can give misleading results in certain scenarios [15]. In [15] we thus provide a generalized framework for the analysis of two-hop single unicast routing that is valid for any mobility scenario that is statistically stationarity, and proceed to show how the framework captures various commonly-used mobility models such as random waypoint, random-walk-on-grid, etc. In this paper, we seek to extend this past work to multicast communication scenarios and arrive at performance bounds for the steady-state throughput.

Though network coding has previously been suggested in the context of ICMANETs in works such as [6], [16], its impact on multicast has hardly been studied. Again, the analysis presented in [16] does not involve performance at steady-state but involves the latency performance of an isolated burst of packets. It is noted that Karande *et al* have shown in [17], [18] that Network Coding does not change the order of throughput in a stationary ad-hoc network, since the same can be achieved by simple store and forward methods employing multipoint transmission and reception. Hence, we have aimed at understanding the performance benefits of multicast with network coding (specifically RLC i.e., Random Linear Coding) in an ICMANET setting. Since multicasting without network coding has not been delved deeply in the context of ICMANETs and involve several issues to be settled [19], we use a simple custodial transfer scheme for comparison.

III. Network Models

A. Definitions

Whenever two nodes are within communication range of each other, we say that a "contact" has occurred between them. However, they may only communicate when a "link" exists between them. The rules for establishing a link such that contention can be resolved between nodes is described later in this section.

Throughout the paper, we assume the following communication scenario: Several relay nodes are deployed in a confined region, which then move independently according to a certain mobility model. We assume that the nodes are identical i.e., they have the same storage buffer size and communication range. These nodes are designated as "relay" nodes and are n in number with a buffer space of B packets each. The network also includes m mobile destination nodes served by a single mobile source which have unlimited storage capacity. Further, nodes may have a certain finite communication range within which it is able to send or receive packets from another node. We perform analysis in discrete time i.e., time is sliced up into several epochs. This assumption is made for clarity and ease of notations, and does not cause any loss of generality. Further, a small buffer size is justified since devices use less memory space for communication, though they might have larger storage capacity.

B. Mobility Model

We assume that the n relay nodes, the source node, and the m destination nodes move independent of each other, but the statistical model for mobility is the same. The underlying mobility model exhibits "statistical stationarity". This means that the probability distribution of the "state" of a node's motion converges over time to a fixed distribution. Though this is critical to our analysis, it is not an unreasonable assumption. It is well-known that a mobility model is meaningful for network evaluation only if it exhibits stationarity [20]. In particular, it is obvious that the random-walk, the random-waypoint model, and their variants are known to be stationary under proper choice of parameters. The mobility of any node v is denoted by a random process $\chi_v(t)$, which at each instant t follows a probability distribution on the state-space $\mathcal{S}_{mob}$ of the given mobility model. For clarity, it is assumed that the state-space $\mathcal{S}_{mob}$ is discrete. Each state may include information such as position, velocity, and waypoint location, etc. Typically, one can describe the state transitions as a linear relationship by means of a transition function $\boldsymbol{\Psi_{mob}}(\cdot)$ (given by the model) as follows. Let $\mathbf{p}(t)$ be the probability distribution of the node's mobility state at time t. Also, let the mobility model have a memory of m' time-steps, where m' is a positive integer. Then, $\mathbf{p}(t+1) = \boldsymbol{\Psi_{mob}}\left[\mathbf{p}(t), \mathbf{p}(t-1), \cdots, \mathbf{p}(t-m')\right]$ for $t > m'$. For example, in case of the random-walk mobility model, $\boldsymbol{\Psi_{mob}}(\cdot)$ simply involves the multiplication of $\mathbf{p}(t)$ by a constant state-transition matrix to obtain $\mathbf{p}(t+1)$. Since the mobility model is assumed to be stationary, it has a unique steady-state probability distribution, $\boldsymbol{\pi}_{mob}$ which satisfies $\boldsymbol{\pi}_{mob} = \boldsymbol{\Psi_{mob}}\left[\boldsymbol{\pi}_{mob}, \cdots, \boldsymbol{\pi}_{mob}\right]$.

C. Link Model and Contention Resolution

We assume a loss-free channel on which a single packet can be transmitted at each epoch. This assumption keeps the discussion simple and focused on issues relevant to this work, though a further extension to relax this can be made easily, as it will be clear from the analysis. We assume that collision is avoided by debarring the nodes within the transmission range of any node sending/receiving packets, from sending/receiving packets on another link.

In addition to the source node transmitting packets over links established with relay nodes and the relay nodes doing the same to the destination, we allow for relay-to-relay communication in our schemes. However, whenever a source node is within the communication range of one or more destination nodes, it establishes a link with one of them with equal probability, if possible. Otherwise, the source/destination node establishes a link with one of the relay nodes within its range (if possible). Last in the order of priority is relay-to-relay communication, wherein all relay nodes that are not within the transmission range of a sender/receiver tries to establish a link with one of the relays in its range randomly with equal probability.

IV. Multicast Routing Protocols

We now describe the routing protocols for multicast that we compare in this paper. It is noted that under both schemes, we need to allow for relay-to-relay communication so that we are not comparing two schemes with poor performance. In the first case, multihop routing makes efficient use of source-to-relay contacts so that replication of packets happens internally and hence the source itself requires each packet to be transmitted only once. In the second case, multihop routing helps to decrease the chance of having redundant packets by increasing diversity.

A. Network-Coded Multicast

We employ RLC coding identical to [6] in order to improve steady-state throughput under multicast. Under this scheme, packets are considered to be vectors in a finite field of desirably large size, say $GF(2^{n_c})$. A relay node, on receiving a packet from the source (or a relay node) makes B scaled versions of the same with coefficients chosen from $GF(2^{n_c})$ and aggregates them onto the contents of its buffer. Whenever a packet is to be sent by the relay to one of the m destinations (or another relay) the former creates a random linear combination of the B entries in its buffer and sends the resulting packet. Again, the coefficients are chosen from $GF(2^{n_c})$. When a relay-to-relay link occurs, one of them is randomly chosen as the sender and a linear combination of its contents are sent to the other. Here, relay-to-relay transmissions is only possible with a blind strategy since it is impossible to keep track of the relay nodes' measure of innovativeness with respect to all the other relays and destinations. Throughout this paper, we assume that the relay nodes never need to delete the contents in its buffer. Also, the destinations do not need to recover the original packets, but are satisfied if they receive a packet that is linearly independent from all the packets received in the past.

B. Simple Custodial Multicast

The comparison scheme used in the paper is a version of custodial transfer modified to suit multicast [19]. In this scenario, each packet is assumed to be uniquely identifiable. On linking with the source, a relay node, if it has an empty buffer space, will accept a packet and assign a counter to it, initialized as m. It is also assumed that when a relay node links with one of the destinations, the latter identifies those packets from the former's buffer that it has not yet received, so that the former sends one of them at random if available. Before the beginning of the next epoch, the relay node decreases the counter associated with the transmitted packet by 1. Packets which have a counter value of zero are discarded before the beginning of next epoch, making room for more incoming packets.

Whenever a link occurs between two relay nodes, they make each other's counter totals known to the other in addition to the IDs of the packets in their buffers. Now, a back-pressure policy is employed to determine which node will send packets. Whichever node has a total counter value higher than the other by at least two will be the sender, and the other one will be the receiver. Then, a packet is sent if the receiver has an empty space in its buffer. Let us say that the packet sent had a counter value of k in the sender's buffer. At the end of the transfer, the sender will update this counter to $\lceil \frac{k}{2} \rceil$ and the receiver will accept the packet with $\lfloor \frac{k}{2} \rfloor$ as the counter for its copy of the same packet. Additionally, if a copy of the same packet is present already in the receiver's buffer, the latter combines both and adds on the value $\lfloor \frac{k}{2} \rfloor$ to the index in the copy. Hence, this protocol aims at redistributing the transmission load among the relay nodes.

V. Analysis of Network-Coded Multicast

From the description of the network-coded scheme, it is clear that obtaining the steady-state performance of this scheme for the ICMANET network model is a cumbersome task. This is because in order to find whether a received packet is innovative, the destination node needs to keep track of all the packets received in the past and has to perform Gaussian elimination on an ever-increasing rank matrix. Hence, only an approximate solution can be obtained by decoding a burst of p packets such that p is sufficiently large. However, this does not help us determine whether the performance obtained thus is an upper bound or a lower bound. Addressing these challenges, we provide the following analysis methodology making use of Markov Chain techniques. The analysis provided thus gives two Markov chains whose steady-state distribution is then solved for iteratively, which in turn determines the throughput performance. In addition, we will show that one of the chains obtained actually provides an upper bound and the other provides a lower bound. In addition, the solution for steady-state distributions can be computed in negligible time for a range of design parameters as the complexity of these chains is manageable and scalable.

A. Overview of the Concept

We first identify an embedded Markov-Chain which describes the dynamics of communication and compute the throughput from its steady-state distribution, by identifying certain "desirable" states of the network. The analysis is further facilitated by a novel technique called "chain collapsing" [15], [21], drastically reducing the complexity of the underlying chain at the cost of a few additional computations. It can be shown that the analysis can be conducted from the standpoint of a single relay v and a particular destination node d.

B. Notations

Relay nodes in the network is identified by a unique integer, $v \in \{1, 2, \cdots, n\}$. For a Markov Chain $X(t)$ with state-space Ω, the steady-state probability of any state $x \in \Omega$ is denoted by $\pi(x)$. The vector $\boldsymbol{\pi}$ denotes the steady-state distribution for the entire state-space $\mathcal{S}$.

C. State-space Description and Throughput

The state of any node v at time t in the network is an ordered pair consisting of mobility-state and its buffer occupancy, as $(\chi_v(t), b_v(t))$. Unlike in our past work [15], the buffer occupancy here refers to the number of innovative packets (i.e., the "virtual" occupancy) w.r.t. the particular destination node picked for the analysis, and hence does not represent the physical state of the buffer, which is always full. We define the state of the entire network as the $2(n+1)$-tuple $\mathcal{Y}(t) \triangleq (\chi_1(t), \cdots, \chi_n(t), b_1(t), \cdots, b_n(t), \chi_s, \chi_d)$. Here, χ_s and χ_d are the mobility states of the source and the destination. We then define the achievable throughput as the *expected rate at which packets are transferred from s to d when the network is in steady-state*. In other words if $\mathcal{N}_{s,d}(\tau)$ packets are transmitted from s to d in time τ, the throughput capacity is given by the relation $\mathcal{C}_{s,d} = \frac{\mathcal{N}_{s,d}(\tau)}{\tau}$.

D. Construction of the Occupancy Matrix and the Complementary Occupancy Vector

Let $[n] \triangleq \{1, 2, \cdots, n\}$. Let $P(i, j)$ be the packet contained in memory-location j of relay i, and let $P(i, j) = \sum_{k=1}^{l} a_k M_k, i \in [n], j \in [B]$, form some vectors $M_1, \cdots, M_k$, where a_k are coefficients in the chosen finite field $GF(2^{n_c})$.
Let $\mathcal{V}(S) \triangleq \text{span}\{P(i, j) | i \in S, j \in [B]\}$ for all $S \subseteq [n]$.

Definition 1: For any two subsets of relay nodes $U, W \subseteq [n]$, we define the *occupancy* of U w.r.t. W as:

$$\Phi(U, W) \triangleq \dim(\mathcal{V}(U)) - \dim(\mathcal{V}(U) \cap \mathcal{V}(W)).$$

We define the *occupancy matrix* for the entire network to be the map $\Phi : 2^{[n]} \times 2^{[n]} \to \mathcal{N}$, where $\mathcal{N}$ is the set of natural numbers and Φ is defined as above.

Definition 2: The *complementary occupancy vector* for our network model is defined as the map $\Psi : 2^{[n]} \to \mathbb{N}$, where $\Psi(S) \triangleq \Phi(S, S^c)$ for any $S \subseteq [n]$, and $S^c \triangleq [n] \setminus S$.

We now need to discuss how the occupancy states are updated in the above setup. This would complete the description of the entire chain for the network. We first define the following operation on the complementary occupancy vector:

Definition 3 (Augmentation): "Augmenting the S-entry w.r.t. i" of the complementary occupancy vector for any given $S \subseteq [n]$ and $i \in S$ consists of the following recursive operation:

a. If $b_S = B\,|S|$, stop.
b. Add 1 to b_S, and (c.)
c. If $|S| = 1$, stop. Otherwise, for every $j \in S$ such that the augmentation results in $b_S - b_{S\setminus\{j\}} > B$, augment $S \setminus \{j\}$ w.r.t. i.

The vector b_S for all $S \subseteq [n]$ is said to be "S-augmentable" if the situation (a.) never occurs during the course of the augmentation.

We can now determine how the occupancy vectors are updated for each of the following cases:

- *Packet arrival:* If node i establishes a link with the source, then augment each $\{i\} \cup S$-entry in the order of cardinality of the S's.
- *Packet delivery:* If node i establishes a link with the destination, then decrease each non-zero b_S such that $i \in S$ by 1.
- *Relay-to-relay interaction:* If node i establishes a link with node j and i transfers a packet to j, then decrease every $b_{S'}$ such that $i \in S'$ and $j \notin S'$ by 1 if $b_{\{j\}^c} - b_{\{i,j\}^c}$ was non-zero to begin with.

It can be shown that the above set of operations always result in a feasible set of b_S. Moreover, it traces the exact occupancy situation of the RLC-encoded relaying scenario. Thus, we can update the states in the chain consistently.

E. The "Stream Separation" Argument

As described previously, throughput is defined from the standpoint of the rate at which linearly-independent packets are received at the destination. In other words, if a received packet is a linear combination of the past history, it does not contribute to the throughput. In other words, for $i \neq j$ if a packet is delivered by a relay v_k to destination D_i, it does not affect the number of innovative packets in $v_k's$ from the standpoint of D_j. Hence, there is no interaction among the m different streams that the node v_k is serving. To explain further, it is as if the relay v_k has m different buffers of size B each dedicated to each of the destinations. Additionally, the throughput contributed by any relay node, on the average, will be equally distributed among all the m destinations. Hence, we can conduct our analysis from the standpoint of a particular destination node served.

F. Upper Bound and Lower Bound Chains

From the Markov-Chain setup described before, we construct two simpler processes (or "chains") $\mathcal{Y}_{\mathcal{U}}(t)$ and $\mathcal{Y}_{\mathcal{L}}(t)$ that can be accurately described with low complexity and solved. In addition, we show that $\mathcal{Y}_{\mathcal{U}}(t)$ leads to an upper bound on the throughput performance and that $\mathcal{Y}_{\mathcal{L}}(t)$ leads to a lower bound. We observe that if two relay nodes have virtual occupancies of b_1 and b_2 corresponding to a particular destination, they do not necessarily have a total of $b_1 + b_2$ innovative packets for the destination served, since they may have exchanged packets before. In the above setup, we would need additional indices that capture such inter-dependencies between the buffer occupancies to describe the process $\mathcal{Y}(t)$ accurately. Since this is cumbersome, we construct two simpler processes (or "chains") $\mathcal{Y}_{\mathcal{U}}(t)$ and $\mathcal{Y}_{\mathcal{L}}(t)$ that can be accurately described. In addition, we will show that $\mathcal{Y}_{\mathcal{U}}(t)$ leads to an upper bound on the throughput performance and that $\mathcal{Y}_{\mathcal{L}}(t)$ leads to a lower bound. In the new processes, the buffer-occupancy indices in the state variable are constructed to be completely independent of each other, thus avoiding correlations. Thus, if relay A has occupancy of b_1 and relay B has occupancy of b_2, then both relays can together contribute exactly b_1+b_2 innovative packets for the destination.

Upper Bound Chain: The new process $\mathcal{Y}_{\mathcal{U}}(t)$ is constructed thus: whenever relay A sends a packet to relay B, we retain the virtual occupancy of A to be the same and increase that of B by 1. Clearly, this overestimates the total innovativeness with respect to all destinations in the network, and hence leads to an upper bound on the throughput.

Lower Bound Chain: The new process $\mathcal{Y}_{\mathcal{L}}(t)$ is constructed thus: whenever relay A sends a packet to relay B, we reduce the virtual occupancy of A by 1 and increase that of B by 1. Clearly, the network-coded schemes performs better than this, as the innovativeness is always underestimated for the sending node.

Having constructed the new processes $\mathcal{Y}_{\mathcal{U}}(t)$ and $\mathcal{Y}_{\mathcal{L}}(t)$ thus, we now proceed to obtain an embedded Markov Chain of low-complexity in lines similar to [15].

VI. Obtaining Embedded Markov Chains

The raw processes $\mathcal{Y}_{\mathcal{L}}(t)$ and $\mathcal{Y}_{\mathcal{U}}(t)$ have $\Theta\left(B^n|\mathcal{S}_{mob}|^{n+2}\right)$ states each. From these large processes, we obtain the corresponding low-complexity embedded Markov chains in the following manner:

As in [15], we can again view the state of the processes from a single relay-node's perspective, compute the throughput due to that node, and scale it up by n to obtain the total throughput. We now group the following subsets of states for both processes:

Let us consider one particular relay node v for this discussion:

- *S-group and D-group subsets:* S_l, with $1 \le l \le B$ is the set of possible network states wherein the *most recent* link that node v was involved in was with the source, resulting in l packets in the buffer after communicating with the latter. Similarly, define $D_{l'}$, for all $B-1 \ge l' \ge 0$ to account for communication with the destination.
- *E-group and F-group states:* F is the set of possible network states wherein the most recent link that node v was involved in was with the source, but v was unable to communicate with the latter due to lack of enough buffer space (i.e., Full/saturated buffer condition). Similarly, define E to account for the case when a link is established with the destination when the buffer-occupancy of v is zero.
- *R-group states:* R_{l_1,l_2} is the set of states such that the last contact that node v had was with a relay node v'. Moreover, v' and v had l_1 and l_2 packets respectively in their buffers before they exchanged any packets.

We can now define two embedded Markov chains corresponding to $\mathcal{Y}_{\mathcal{L}}(t)$ and $\mathcal{Y}_{\mathcal{U}}(t)$ from the perspective of node v based on these subsets. In the above enumeration, note that there are B subsets each in the S- and D- groups, and $(B+1)^2$ subsets in the R-group. Additionally, we have two other subsets: E and F. Hence, there is a total of $(B+1)^2 + 2(B+1)$ subsets. By using Theorem 2 in [15], we can obtain equivalent "collapsed chains" that has just as many states. Let us call these two new processes the $\Gamma_{v;L}$- and $\Gamma_{v;U}$- chains. Our aim is to find an efficient way to compute the steady-state probability distribution of the states in Γ_v chains (which we denote by the vector $\boldsymbol{\pi}$) so that one can avoid the cumbersome simulations involving several nodes.

We note that both the Γ_v-chains consists $(B+1)^2$ extra states corresponding to relay-to-relay packet exchanges in addition to the states in the embedded chain corresponding to the two-hop relay protocol discussed in [15]. Both chains have a similar structure after the inclusion of R-states is shown in Fig. 1, where the states that are common with the corresponding two-hop chain are shown in solid bubbles. In this figure, all the possible transitions from various states (or groups of states) is shown. The internal structure of the R-chains vary according to their definitions. The key differences in structure between $\Gamma_{v;L}$ and $\Gamma_{v;U}$ are shown in Figs. 2(a) and 2(b) respectively. In the first case, transition from $R_{2,4}$ to D_2 is possible, as this corresponds to the scenario where relay node v has occupancy of four initially, delivers one packet to the other relay node with initial occupancy of two, and sees a reduction in its occupancy, and hence has an occupancy of 3 during the next transition before linking with the destination node. However, since we do not reduce the occupancy under the same link scenario in the upper-bound case, the next link with the destination leads to the state D_3 and not to D_2.

A. Computation of Transition Probabilities and Steady-State Distributions for the Γ_v-Chains

We will compute the individual transition probabilities between various states in the chain in terms of a single quantity

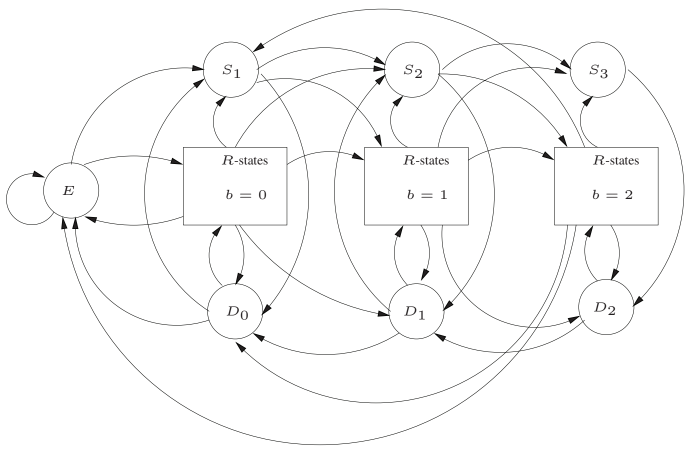
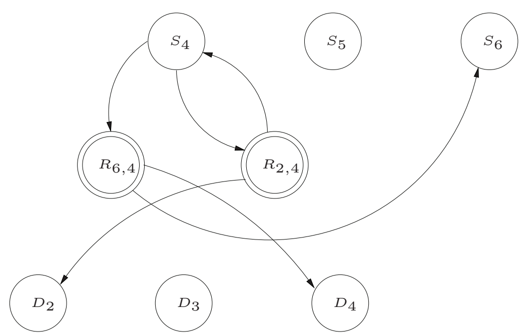
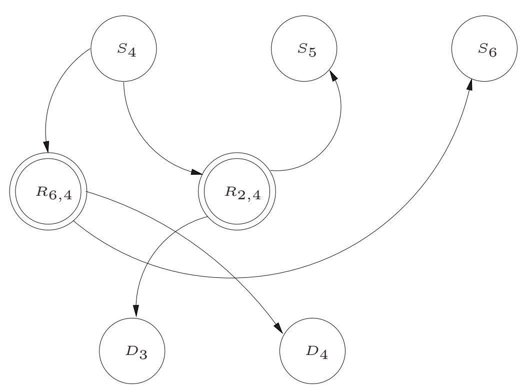


Fig. 1. General structure of the upper-bound- and lower-bound- embedded chains.



(a) Possible Transitions for the lower bound chain



(b) Possible Transitions for the upper bound chain

Fig. 2. Differences in the structures of the upper-bound and lower-bound chains

$\alpha(n)$. Again, we note that only one of the m destination nodes is in consideration for this discussion. We define $\alpha(n)$ in the following manner: given that a particular relay node u is currently has a link with the source/destination node or a relay-node, $1 - \alpha(n)$ is the probability that u will establish a link with the same again before establishing one with any other node in the network. Next we note that since the node mobilities are not correlated, the probability that the node u will link again with a some other node u' different from the node which shared the last link, is exactly given by $\frac{\alpha(n)}{n}$ for each u'. In other words, since there are $n+2$ nodes, including the source and the destination, there are n nodes other than u and u' themselves and each one of them is equally likely to win the next link for node u. We will later proceed to find that $\alpha(n)$ itself is a function of (i) Mobility parameters, (ii) Networking parameters, and (iii) Contention protocol, and its computation of the same in terms of these parameters will follow.

Next, we define the following probabilities for the Γ_v-chains:

- Given that v currently has a link with the source, the probability that its next link will be with the source again is given by p_{ss}, and the probability that its next link will be with the destination node is given by p_{sd}. Similarly, one can define p_{dd} and p_{ds} in the same manner.
- Given that v currently has a link with some relay node, the probability that the next link will be with the same/another relay node is given by p_{rr}. Similarly, we can define p_{sr}, p_{dr}, p_{rs}, and p_{rd}.

We now proceed to examine individual transition probabilities in the Γ_v-chains. Let us define Ω as the entire set of $2(B+1)$ S, D, E, and F-states. For any two states X and X' such that $X \in \Gamma_v$ and $X' \in \Omega$ such that it is possible to reach the latter directly from the former, the probability of such a transition does not depend on the current buffer-occupancy of the relay node v.[1] Hence, these probabilities will be the same as p_{ss}, p_{sd}, p_{dd}, p_{ds}, p_{rd}, or p_{rs}, as the case may be.

Due to the symmetry of our network model and due to independent mobility, one can easily verify that these probabilities are given in terms of $\alpha(n)$ by the following equations:

$$\begin{aligned}
p_{ss} &= p_{dd} = 1 - \alpha(n) \\
p_{rr} &= p_{ss} + \frac{n-2}{n}(1 - p_{ss}) \\
p_{sd} &= p_{ds} = p_{rd} = p_{rs} = \frac{\alpha(n)}{n} \\
p_{sr} &= p_{dr} = \frac{n-1}{n}(1 - p_{ss})
\end{aligned}$$

The transitions into the R-states are slightly more complicated. We discuss these transitions in the proof of the following theorem, summarizing the construction of the entire embedded Markov Chain for both cases. Here, we only describe the result for $\mathcal{Y}_{\mathcal{L}}(t)$. The procedure is similar for the $\mathcal{Y}_{\mathcal{U}}(t)$-chain with a obvious minor changes.

Theorem 1: The steady-state distribution of the states in the collapsed embedded Chain $\Gamma_{v;L}$ is given by the following system of equations. For the sake of convenience, we define $\pi\left(R_{l_1,l_2}\right)$ to be zero if either l_1 or l_2 lies outside the interval

[1]This is due to the fact that neither mobility nor the contention protocol depends on B. The case in which the contention protocol takes into consideration buffer occupancies of the relay nodes is avoided for clarity, but is similarly possible with minor modifications to the analysis.

$[0, B]$ in these equations:

$$\begin{aligned}\pi(E) &= p_{dd}\{\pi(E)+\pi(D_0)\}+p_{rd}\pi(R_{0,0})\\ &+ p_{rd}\left\{\frac{1}{2}\sum_{i=0}^{B-1}\pi(R_{i,1})+\frac{1}{2}\sum_{i=1}^{B}\pi(R_{i,0})\right\} \qquad (1)\\ \pi(F) &= p_{ss}\{\pi(F)+\pi(S_B)\}+p_{rs}\pi(R_{B,B})\\ &+ p_{rs}\left\{\frac{1}{2}\sum_{i=0}^{B-1}\pi(R_{i,B})+\frac{1}{2}\sum_{i=1}^{B}\pi(R_{i,B-1})\right\} \qquad (2)\end{aligned}$$

$$\pi(D_k) = \begin{cases} p_{dd}\pi(D_{k+1})+p_{sd}\pi(S_{k+1}) +p_{rd}\left\{\frac{1}{2}\sum_{j=0}^{B-1}\pi(R_{j,k+2}) + \sum_{j'=1}^{B}\pi(R_{j',k})\right\}, 0\le k\le B-2 \\ p_{sd}\{\pi(F)+\pi(S_B)\} +p_{rd}\left\{\frac{1}{2}\sum_{i=0}^{B-1}\pi(R_{i,B})+\pi(R_{B,B}) + \frac{1}{2}\sum_{i=1}^{B}\pi(R_{i.B-1})\right\}, k=B-1 \end{cases}. \quad (3)$$

$$\pi(S_k) = \begin{cases} p_{ds}\{\pi(E)+\pi(D_0)\} +p_{rs}\left\{\frac{1}{2}\sum_{i=1}^{B}\pi(R_{i,0})+\pi(R_{0,0}) + \frac{1}{2}\sum_{i=0}^{B-1}\pi(R_{i,1})\right\}, k=1 \\ p_{ss}\pi(S_{k-1})+p_{ds}\pi(D_{k-1}) +p_{rs}\left\{\frac{1}{2}\sum_{j=1}^{B}\pi(R_{j,k-2}) +\frac{1}{2}\sum_{j'=k-0}^{k}\pi(R_{j',k})\right\}, 1<k\le B \end{cases}. \quad (4)$$

For any $0 \le l_1 \le B, \varphi_{l_1,l_2}^{-1}\pi(R_{l_1,l_2})$

$$= \begin{cases} p_{dr}\{\pi(E)+\pi(D_0)\} +p_{rr}\left\{\pi(R_{0,0})+\frac{1}{2}\sum_{i=0}^{B-1}\pi(R_{i,1}) +\frac{1}{2}\sum_{i=1}^{B}\pi(R_{i,0})\right\}, l_2=0 \\ p_{sr}\{\pi(F)+\pi(S_B)\} +p_{rr}\left\{\pi(R_{B,B})+\frac{1}{2}\sum_{i=0}^{B-1}\pi(R_{i,B}) +\frac{1}{2}\sum_{i=1}^{B}\pi(R_{i,B-1})\right\}, l_2=B \\ p_{sr}\pi(S_{l_2})+p_{dr}\pi(D_{l_2})+p_{rr}\left\{\frac{1}{2}\sum_{i=0}^{B-1}\pi(R_{i,l_2+1}) +\frac{1}{2}\sum_{i=1}^{B}\pi(R_{i,l_2-1})+\pi(R_{B,l_2})\right\}, 0<l_2<B \end{cases} \quad (5)$$

where $\varphi_{l_1,l_2} \triangleq \dfrac{\pi(R_{l_2,l_1})}{\sum_{j=0}^{B}\pi(R_{j,l_1})}$ for any $0 \le l_1, l_2 \le B$.

Proof: The proof for the steady state equations (1)-(4) follow from the previous discussion in this section. We only need to analyze the transitions into the R-states. Given that the current link for node v is with a source, destination, or some relay, the probability that the next link is with any of the $n-1$ other relays is given exactly by the quantity p_{rr}. Now in order to determine the probability that the relay node corresponding to the new link has exactly l_1 packets in its buffer, we can use the chain collapsing principle. Equivalently, we only need to determine the "subset-averaged" probability distribution of the buffer occupancy of the any relay node every time v comes into contact with the same. By symmetry, we can say that this distribution, denoted as φ above is exactly the same as the distribution of node v every time it comes into contact with a relay node that has occupancy l_1. We call φ as the "joint buffer-occupancy" distributions for the chain Γ_v. Hence, the expression for φ is exactly as given in (5). Knowing this, we can then compute the steady-state probability of each R-state by carefully examining the possible previous states for v in Γ_v. ■

The analysis of steady-state for the Γ_v chain would be complete once we determine the unknown parameter $\alpha(n)$ in terms of networking and mobility parameters. Doing so would enable us to study scaling laws with respect to network size (n), node-density, mobility characteristics, etc. The relevant analysis follows in the next subsection.

B. Contention Analysis and Mobility Parameters

The computation of $\alpha(n)$ in closed form necessitates us to look into the effect of node-to-node contention (which has not been considered previously). We derive the same in the following manner: establish the dependence on the contention protocol first, and finally establish the dependence on the mobility characteristics. The first step is accomplished with the aid of yet another embedded chain, defined on the following sub-sets of states, defined with respect to the particular relay-node v. Let U be any node in the network other than v (source, destination, or another relay node):

- U_W: Most recent contact occurred with U, and contention was won
- U_L: Most recent contact occurred with U, and contention was lost
- X_W: Most recent contact occurred with some other node than U, and contention was won
- X_L: Most recent contact occurred with some other node than U, and contention was lost

Once again, this is a variation on the contention chain described in [15]. The analysis of this chain is similar, and the final result is given below:

$$\alpha(n) \quad = \quad \frac{\alpha_0(n)}{\frac{n+1}{n}\alpha_0(n)\beta_c + (1-\beta_c)}.$$

where $\alpha_0(n)$ is defined in a similar manner as $\alpha(n)$ with the exception that only a contact has to occur. Hence, $\alpha_0(n)$ depends primarily on the mobility characteristics and has no dependence on the contention model. Here, β_c is the average probability of v losing the contention phase, and an expression for the same is provided in [15]:

$$\begin{aligned} 1-\beta_c \quad &= \quad \left(1-\frac{1}{E\left[T_0\right]}\right)\sum_{k=0}^{n-1}\sum_{\mathbf{x}}\frac{\pi_{spt}(\mathbf{x})}{k+1}\binom{n-1}{k}\pi_{spt}^{k}(\mathbf{x}') \\ &\times \quad \left\{1-\pi_{spt}(\mathbf{x}')\right\}^{n-1-k}. \qquad (6)\end{aligned}$$

where $E\left[T_0\right]$ is the average inter-contact interval and π_{spt} is the steady-state spatial distribution of the underlying mobility model.

Finally, for most mobility model, we can obtain $\alpha_0(n)$ in terms of a good approximation (from [15]):

$$\alpha_0(n) \approx \frac{E\left[T_0\right]}{E\left[T_\infty\right]}\frac{n}{n+1}. \qquad (7)$$

where $E\left[T_\infty\right]$ is the average waiting time for a particular contact to occur, starting from steady-state.

In order to complete the analysis, one needs to determine the steady-state probabilities of all the states in this chain. This is done by solving the equations (1)-(5). However, we note that the steady-state equations for the R-states (5) are non-linear. Hence, it is impossible to obtain closed-form solutions for the throughput. Nevertheless, (1)-(5) can be solved by iterative methods. Having obtained the steady-state distribution of $\Gamma_{v;L}$ chain thus, we can compute the throughput (in packets per epoch), for the $\mathcal{Y}_{\mathcal{L}}(t)$ process, as a contribution of all the n relay nodes as follows:

$$\mathcal{C}_{L;s,d} \quad = \quad \frac{1}{E\left[T_0\right]}\frac{n(1-\beta_c)}{E\left[T_0\right]}\frac{\sum_{j=0}^{B-1}\pi_L\left(D_j\right)}{\pi_L\left(E\right)+\sum_{j=0}^{B-1}\pi_L\left(D_j\right)}. \qquad (8)$$

Subscript L in the steady-state distribution indicates that the solution is from the corresponding $\Gamma_{v;L}$ chain. The above expression follows from the fact that the throughput is given by the ratio of the total steady-state probability of the "desirable" states (in the Γ_v chains, these are the D-states) to the total steady-state probability of all states where the destination node is linked with (i.e., E- and D-states), times the frequency of establishing a link with the destination which can be computed as $\frac{(1-\beta_c)}{E[T_0]}$. The first term in the throughput indicates the contribution of direct source-to-destination contacts.

Similarly, we obtain the throughput estimated by the process $\mathcal{Y}_{\mathcal{L}}(t)$. Finally, the actual throughput of multicast with RLC network coding is given by $\mathcal{C}_{L;s,d} \le \mathcal{C}_{s,d} \le \mathcal{C}_{U;s,d}$.

To summarize, we have obtained a scalable iterative technique for bounding the throughput performance of network-coded unicast. In general, this queuing-theoretic model converges within 10-20 iterations. In contrast, simulating the exact network for a given n takes a few million epochs for the system to reach steady-state. In addition, simulations are not quite scalable for certain parametric choices.

VII. Simulation Results

In order to verify our analysis, we simulated the above IC-MANET network model under the random-waypoint mobility model. Relay nodes, 50 in number, were deployed on a square region of size $5km$ by $5km$. Each node was assumed to have a radio range consisting of a circular disc of radius $250m$. The velocities of nodes were chosen randomly according to uniform distribution, between $4kmph$ and $9kmph$. Waypoints were randomly chosen from a uniform spatial distribution across the entire deployment region. The throughput performance of such a network at steady-state was obtained for different choices of buffer sizes (B) and for different numbers of destination nodes served (m). We simulated both the network-coded scheme as well as the simple custodial-multicast scheme, and compared the results with the bounds obtained by iteratively solving the two Markov Chains described in the previous section. For the network-coded scheme, we used a Galois field of size 397. A prime number was chosen rather than a power of two, since operations are easier in the former case. For both schemes, we simulated 100000 epochs, and repeated the same for 50 different initial conditions.

In the first plot shown in Fig. 3, the per-node buffer size in the network was varied from 8 to 40 packets, while 10 destinations were served. Under this regime, it is seen that the network-coded scheme offers considerable improvement on the throughput. However, it was observed that the percent improvement in throughput goes down as we increase the buffer sizes further, and they both tend to saturate near the same level.

This is due to the fact that the benefits of network coding are contributed primarily by its buffer-management strategy.

In the second plot shown in Fig. 4, the buffer sizes were kept constant at 16 packets per node while m was varied from 5 through 20. Clearly, the higher the number of destinations served, the simple custodial scheme degrades drastically in terms of per-destination-node throughput. However, it was observed that the network-coded scheme hardly diminished in performance.

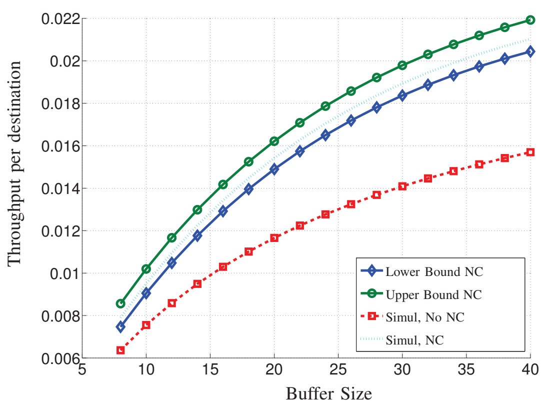


Fig. 3. Simulation results for varying buffer sizes

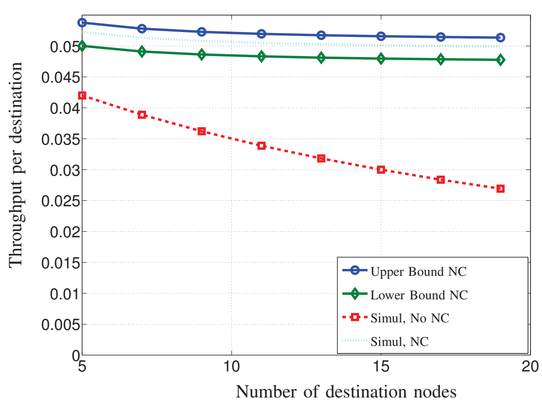


Fig. 4. Simulation results for varying number of destinations

VIII. Conclusions

In this work, we developed a novel methodology to bound the performance of network-coded multicast in ICMANETs. We presented a generalized, scalable iterative framework based on Markov-chain analysis that incorporates several considerations such as finite buffers, generalized mobility, and node-to-node contention. Our results show that Network Coding offers significant benefits for multicast especially in the finite-buffer regime vis-a-vis simpler custodial-multicast schemes. The Markov-chain-based methodology we developed provided good upper and lower bounds for performance under network coding. Future extensions that will be considered include an analysis framework for replication-based multicast schemes, of which the custodial multicast scheme presented here is a special case. In addition, there is a need to study the performance of network-coding schemes wherein packets are processed in blocks, rather than on the entire stream. Though block-based network coding is more desirable in practice, it remains to be seen whether the benefits offered by network coding apply to this scenario.

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