Delay Analysis of Disruption Tolerant Networks with Two-Hop Routing in a Finite-Buffer Regime

Nima Torabkhani and Faramarz Fekri
School of Electrical and Computer Engineering
Georgia Institute of Technology, Atlanta GA 30332, USA
Email: {nima, fekri}@ece.gatech.edu

***Abstract*—We consider disruption tolerant networks (DTNs) wherein a direct communication path from a source to a destination via multiple hops does not exist due to both mobility and sparseness of the nodes. Hence, mobile nodes will deliver messages from source to destination using a "store, carry, and forward" strategy. In this paper, our goal is to analytically study the packet latency in such networks for a two-hop unicast scenario with Bernoulli packet arrivals at the source. We exploit an embedded Markov chain approach combined with our novel iterative estimation technique to study both network delay and queuing delay. Constraints posed by both the limited node buffer size and contention between nodes for wireless channel are also considered to obtain a more realistic model. Finally, our results are validated using simulations for a random-walk on a two-dimensional grid mobility model.**

I. Introduction

Disruption tolerant networks (DTNs), also referred to as delay-tolerant networks, is an special type of mobile ad-hoc networks. They often used when there is no backbone infrastructure and hence have applications in military networks, vehicular networks, and providing basic network services to rural areas.

Conventional Mobile Ad-hoc Networks (MANETs) rely on the existence of end-to-end paths between source and destination nodes regardless of node mobility. However in DTNs simultaneous end-to-end connectivity is very rare because of the sparseness of nodes in the network. Hence, communication protocols for MANETs perform inefficiently for DTNs. Most of the efficient DTN-based schemes [1], [2], use the "store, carry, and forward" paradigm for message delivery, wherein a source node opportunistically transmits packets upon contacting any other node, and relies on the mobility of these "relay" nodes to deliver the message to a certain destination.

Analytical performance modeling of DTNs has recently drawn a considerable amount of attention [3]–[8]. In many cases, the performance of DTNs have been modeled using Poisson process approximations [5]–[7]. Investigated in [9], a major drawback of this approximation is that assuming Poisson process for contact times does not incorporate the spatial-temporal dependence between contact times of any pair of nodes which is not a realistic assumption in general. Inspired by such shortcomings, in [9], Subramanian *et al.* proposed a generalized framework for throughput analysis of finite-buffer DTNs. The framework uses the embedded Markov chain approach using which the throughput of such networks can be identified by computing certain well-defined characteristic parameters from the mobility model. Further, the problem of throughput analysis in DTNs has been considered for many different communication scenarios and mobility models in [9]–[11], and hence, is well-motivated. However, despite the usefulness and value of such a framework for throughput analysis, it is insufficient for modeling the performance of such networks under different types of traffic, for the following reasons:

- To identify throughput in the previous model, it is assumed that source is always backlogged, *i.e.*, it possesses infinitely many information packets. Consequently, all relay nodes will be as congested as possible. Thus, such an assumption for the source node leads to finding the *maximum* average "network delay" only.
- Since the source is constantly backlogged, it eliminates the necessity of defining queueing delay at the source despite being an important performance parameter.
- The problem of performance analysis of multiple unicast sessions [10] can be useful only when various sources could own different traffic characteristics. In other words, resource sharing protocols will not have a great impact on the performance of the network if all the flow sources are backlogged, and share the network resources such as memory or bandwidth equally.

In this paper, we address the problem of delay analysis for a single unicast session, where a single source node is trying to transmit packets to a single destination using mobile relays. Further, a dynamic queue is assumed for the source with exogenous Bernoulli arrivals of packets. Practical issues such as finite storage space for relays, random contact times, and contention between nodes to access the channel are also considered to make the analysis more realistic. We will use analytical tools such as embedded Markov chain and chain-collapsing idea combined with our novel iterative estimation technique to estimate the steady-state distributions of buffer occupancies for relays and the source. We then illustrate the framework in detail for a random-walk on a grid mobility model and, finally, validate the analysis using simulations.

This material is based upon work supported by the National Science Foundation under Grant No. CCF-0914630.

978-1-4799-1353-4/13/$31.00 ©2013 IEEE

II. Network Model

We consider n identical nodes, referred to as "relay" nodes, and two other nodes, referred to as "source" and "destination" nodes. The nodes are located randomly in grid and moving independently according to the mobility model specified in Section II-B. The relay nodes have the same buffer size of B packets where each packet have a fixed length. However, source and destination nodes have unlimited storage capacity. A discrete-time model is used where at each time epoch, only one packet may be transmitted/received by any node. Further, communication is assumed to be error-free. Addressing this problem in presence of channel erasure can be shown to be a straightforward extension of the current framework, and will not be discussed in this paper.

A. *Packet Arrivals at Source*

A crucial part in modeling latency would be to consider exogenous packet arrivals to the source. We assume, at each time epoch, a single packet arrives at the source with probability λ, *i.e.*, the rate of arrival at the source is λ packets per epoch. In this paper, we only consider mean arrival rates λ less than the throughput capacity of the network. This will guarantee that the queue length at the source remains bounded with high probability, and hence, the network is stable[1]. Consequently, such assumption leads to the boundedness of the average queueing delay at the source. Note that, by choosing λ larger than the throughput capacity, the queue at the source will grow unboundedly, since the network fails to deliver packets with such rate. In conclusion, without loss of generality, we assume that λ is smaller than the network throughput capacity [9].

B. *Mobility Model*

Throughout this paper, we will consider the random walk on a two-dimensional grid as the mobility model. In this model, nodes are randomly moving on a $M \times M$ square grid as shown in Fig. 1. At each time epoch, nodes may remain at the same cell, or move to an adjacent cell with a certain probability. The transition probabilities for the random walk are chosen so that it results in a uniform steady-state spatial distribution, *i.e.*, a node is located in a specific cell with probability $\frac{1}{M^2}$. Hence, we choose the probability of transition to adjacent cells to be $\frac{1}{5}$ and the self-transition probability for each cell will be $1-\frac{\text{No. of adj. cells}}{5}$. As an example, for the cell in the corner, the self-transition probability is equal to $\frac{3}{5}$.

C. *Interference Model*

The communication between two nodes is possible only if they are at the same cell in the grid. All the other nodes in that particular cell are assumed to be silent for the duration of the communication (one epoch). This is to ensure that there is no wireless interference issues such as hidden-terminal and exposed-terminal situations. Moreover, the source/destination node tries to establish a new link at each epoch, for which several relay nodes may contend. To be precise, in each epoch, if the source and destination are at the same cell, they will form a link, otherwise, if the source or destination and multiple relays are located at a cell, a random relay is selected to setup a link with source or destination, respectively. We say that a "contact" occurs between two nodes whenever they are in the same cell, though they may not communicate. If two nodes win the channel contention, a "link" is said to be established between the communicating nodes.

[1]Note that, the queues at relays cannot grow to infinity since they have a finite buffer size.

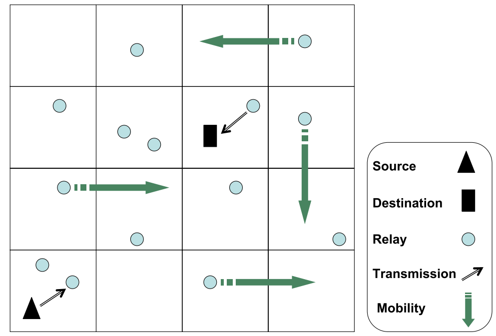

Fig. 1. Network Model

D. *Two-hop Single-copy Routing*

When a relay node with available space in its buffer establishes a link with the source, it accepts a packet if the source has at least one packet,*i.e.* it is non-empty, and retains the packet until a link is established with the destination. As it is shown in Fig. 1, no relay-to-relay communication occurs. In addition, though very rarely, the source and destination may establish a direct link.

III. Markov Chain Analysis

Thoroughly investigated in [9], the full state-space description of finite-buffer DTNs is very large to work with. It is shown that a complex multidimensional Markov chain containing the states of all buffers and node locations has to be analyzed to derive the throughput for such networks. This further compounds with the introduction of new states corresponding to the source queue. To reduce the state-space and simplify the analysis, we will exploit the idea of chain-collapsing as in [9]. Throughout this work, for any $x \in [0, 1]$, we define $\overline{x} \triangleq 1 - x$.

A. *The Idea of Chain-Collapsing*

To simplify the analysis, as a first step, we may try to identify certain symmetries in the network that simplifies the state space. For example, in a scenario where relay nodes are identical, one can view the state of the network from a single relay's perspective. However, the state space still remains very large. Next, we try to address the simplification problem by deriving a new set of "desirable" states from the original state space such that the steady-state probability distributions are preserved. Then, for a particular relay node, we identify all those "desirable" states in which one could track how much time packets spend inside relays/source

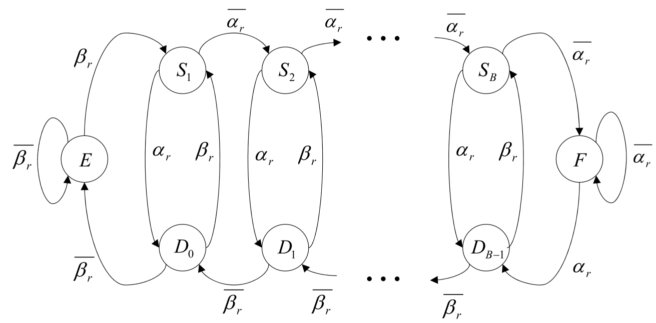

Fig. 2. The embedded Markov chain for a relay node (RMC)

together with certain additional "auxiliary" states to arrive at an "embedded" Markov chain. The idea of chain collapsing enables us to extract only the necessary information from the original Markov chain. In particular, we first need to reduce the performance computation problem to computing the total steady-state probabilities of certain subsets of a well-defined embedded Markov chain. Note that, we are not interested to find individual steady-state probabilities of states within one particular desired subset of states. After finding the appropriate set of states, the rest of the analysis will only include computation of the transition probabilities between the desired subsets followed by computation of their steady-state probabilities.

B. Embedded Markov Chain for a Relay Node

In this section, our objective is to define the states of the embedded Markov chain for a single relay node so that the resulting steady-state probabilities could provide us with sufficient information to approach the problem of delay analysis. Hence, we define the embedded Markov chain for a relay node according to the following subsets of states:

- Let S_i $(1 \leq i \leq B)$ be the set of network states wherein the most recent link that node v had was with a non-empty source, resulting in i packets in the buffer after receiving a packet.
- Similarly, let D_j $(0 \leq j \leq B-1)$ be the set of network states wherein the most recent link that node v had was with the destination, resulting in j packets in its buffer after transmitting a packet.
- Let F be the set of network states wherein the most recent link that node v had was with a non-empty source, but v was unable to accept any packet due to lack of buffer space (*i.e.*, Full buffer state).
- Similarly, let E be the set of network states wherein the most recent link that node v had was with the destination, but v had no packet to transmit (*i.e.*, Empty buffer state).

Given the state transition probabilities for the embedded Markov chain in Fig. 2 (RMC), a closed-form expression for its steady-state probabilities can be easily obtained using

$$
\begin{aligned}
\Pr\{F\} &= \left(\frac{\overline{\alpha_r}}{\overline{\beta_r}}\right)^B \frac{\beta_r}{\alpha_r} \Pr\{E\}, \\
\Pr\{S_{i+1}\} = \Pr\{D_i\} &= \left(\frac{\overline{\alpha_r}}{\overline{\beta_r}}\right)^i \frac{\beta_r}{\overline{\beta_r}} \Pr\{E\},
\end{aligned}
$$

for $i = 0, \ldots, B-1$, and,

$$
\Pr\{E\} = \begin{cases} \frac{1}{2}\left\{1 + \frac{\beta_r}{\overline{\beta_r}} B\right\}^{-1}, & \text{if } \alpha_r = \beta_r \\ \frac{\alpha_r - \beta_r}{\alpha_r + \beta_r}\left\{1 - \frac{\beta_r}{\alpha_r}\left(\frac{\overline{\alpha_r}}{\overline{\beta_r}}\right)^B\right\}^{-1}, & \text{if } \alpha_r \neq \beta_r \end{cases}.
$$

Further, the state transition probabilities for RMC can be obtained using the following lemma.

Lemma 1 *Let α_0 be the probability that a node currently in contact with the source (or destination) will have a contact with the destination (or source) before coming in contact with the former again. Also, let p_c be the average probability that a relay node loses contention on meeting the source/destination node. Finally, let p_e be the probability that source node is empty,*i.e. *have no packets in its queue, when meeting a relay or destination node. Then,*

$$
\alpha_r = \frac{\alpha_0}{\overline{p_e}(2\alpha_0 p_c + \overline{p_c}) + \alpha_0 p_e}, \qquad \beta_r = \overline{p_e}\alpha_r.
$$

Proof: The proof is very similar to the proof of Lemma 4 (see Appendix). ■

The contention failure probability can be easily calculated for the random-walk on a grid mobility model using the following result [9].

Lemma 2 *For a network with n relay nodes and a buffer size of B with a random-walk mobility model which has uniform spatial distribution, the probability that a relay node loses contention when meeting the source/destination (p_c) can be derived from*

$$
p_c = 1 - \frac{M^2}{n}\left(1 - \left(1 - \frac{1}{M^2}\right)^n\right)
$$

Finally, the parameter α_0 in Lemma 1 is characterized for a general mobility model in the following lemma.

Lemma 3 *Let T_0 be a random variable representing the inter-contact duration, and let T_∞ be the random variable representing the waiting time until two nodes meet, given that they are distributed according to the steady-state spatial location distribution. Then we have*

$$
\alpha_0 = \sum_{\tau=1}^{\infty} F_{T_\infty}(\tau) P_{T_0}(\tau),
$$

where $P_{T_0}(\tau)$ and $F_{T_\infty}(\tau)$ are the probability density function of T_0 and the cumulative density function of T_∞, respectively.

Proof: The proof is very similar to the proof of Lemma 5 (see Appendix). ■

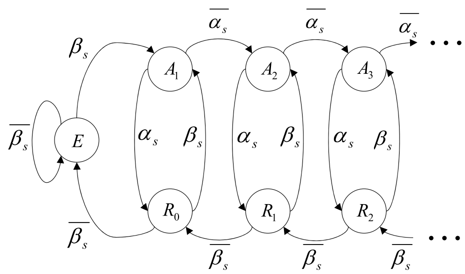

Fig. 3. The embedded Markov chain for the source node (SMC)

C. Embedded Markov Chain for the Source

In this section, our objective is to define the states of the embedded Markov chain for the source node so that the resulting steady-state probabilities could be helpful for the problem of delay analysis. Hence, the embedded Markov chain for the source node is defined according to the following subsets of states:

- Let A_i $(i = 1, 2, \ldots)$ be the set of network states wherein the most recent event at the source is an arrival of a packet, resulting in i packets in the source buffer.
- Also, let R_j $(j = 0, 1, \ldots)$ be the set of network states wherein the most recent event at the source is meeting a non-full relay or the destination node, resulting in j packets in the source buffer.
- Finally, let E be the set of network states wherein the most recent event at the source is meeting a non-full relay or the destination node while the source node is empty and hence no packet is transmitted

Given the state transition probabilities for the embedded Markov chain in Fig. 3 (SMC), a closed-form expression for its steady-state probabilities can be easily obtained using

$$\begin{aligned}\Pr\{R_i\} = \Pr\{A_{i+1}\} &= \left(\frac{\overline{\alpha_s}}{\overline{\overline{\beta_s}}}\right)^i \frac{\beta_s}{\overline{\overline{\beta_s}}} \Pr\{E\}, \quad i \geq 0 \\ \Pr\{E\} &= \frac{\alpha_s - \beta_s}{\alpha_s + \beta_s}.\end{aligned}$$

Moreover, the state transition probabilities of SMC (α_s, β_s) can be derived using the following lemma.

Lemma 4 *Let α_1 be the probability that the source node with a packet currently arriving to its queue, will have a contact with any other node (relay or destination) before having another packet arriving to its queue. Further, let α_2 be the probability that the source node currently in contact with a node (relay or destination) will have an arriving packet to its queue before coming in contact with any other node in the network. Finally, let p_f be the probability that a relay node is full when meeting with the source and is unable to accept any packets. Then, we have*

$$\alpha_s = \frac{\alpha_1 \overline{p_b}}{\alpha_2 p_b + \overline{p_b}}, \qquad \beta_s = \frac{\alpha_2}{\alpha_2 p_b + \overline{p_b}},$$

where, $p_b = \frac{n}{n+1} p_f$.

Proof: See Appendix for a brief sketch of the proof. ■

Finally, parameters α_1 and α_2 can be characterized for a general mobility model using the following lemma.

Lemma 5 *Let S_0 be a random variable representing the inter-arrival time duration of packets at source, and let S_∞ be the random variable representing the waiting time until an arrival of a packet given no information about the previous arrivals. Further, let $T_{\infty,n}$ be a random variable representing the waiting time until a contact with one of $n+1$ relays/destination occur for the source, given that the nodes are distributed according to the steady-state spatial location distribution (uniform for the case of random-walk on grid).Also, let $T_{0,n}$ be a random variable representing the waiting time until source makes contacts with one of the $n+1$ relays/destination nodes, given that source is currently in contact with a relay/destination and the other n nodes are distributed according to the steady-state spatial location distribution. Then,*

$$\begin{aligned}\alpha_1 &= \sum_{\tau=1}^{\infty} F_{T_{\infty,n}}(\tau) P_{S_0}(\tau) \\ \alpha_2 &= \sum_{\tau=1}^{\infty} F_{T_{0,n}}(\tau) P_{S_\infty}(\tau),\end{aligned}$$

Proof: See Appendix for a brief sketch of the proof. ■

D. Iterative Estimation

Thus far, we have developed two different collapsed Markov chains, RMC and SMC, originated from the full state-space of the entire network. In other words, we have observed the desirable states of the network from the point of view of both a single relay node and the source node. However, it is notable that deriving the state transition probabilities for RMC and SMC requires using Lemmas 1, and 4 in which the parameters p_f and p_e are not known in advance. In this section, we will see that these two Markov chains are not only dependent on each other but also closely related. Further, their dependency could lead us into solving both of them using an iterative algorithm.

To find p_f, we need to know the portion of relay-source links during which the relay is full. Using steady-state probabilities of RMC, the following can be shown

$$p_f = \overline{\alpha_0}\ \overline{p_c}\ \frac{\Pr\{F\} + \Pr\{S_B\}}{\Pr\{F\} + \sum_{i=1}^{B} \Pr\{S_i\}}. \tag{1}$$

Further, obtaining the steady-state probabilities of RMC requires having its state transition probabilities by using Lemma 1. Hence, we need to find p_e which is the portion of source-relay/destination links during which the source is empty. Using steady-state probabilities of SMC, the following relation can be obtained

$$p_e = \overline{\alpha_2}\ \frac{\Pr\{E\} + \Pr\{R_0\}}{\Pr\{E\} + \sum_{i=0}^{\infty} \Pr\{R_i\}}. \tag{2}$$

Finally, obtaining the steady-state probabilities of SMC requires having its state transition probabilities by using

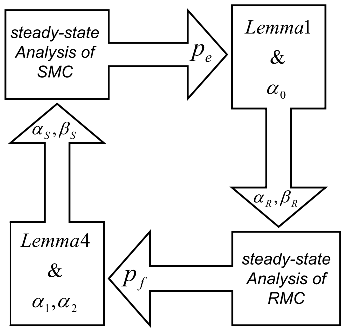

Fig. 4. A graphical presentation of the iterative estimation algorithm

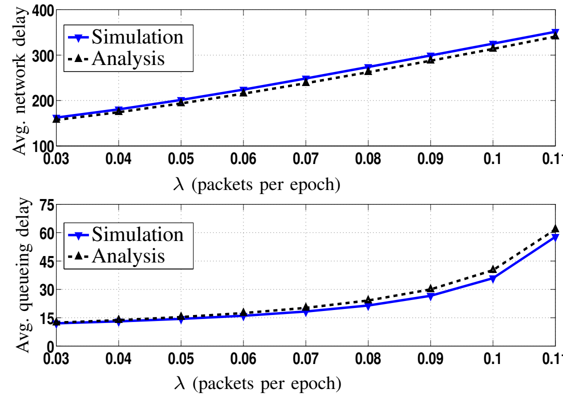

Fig. 5. Variations of average network delay and average queueing delay at the source with mean arrival rate λ for a random walk on a grid mobility model

Lemma 4 and hence knowing p_f. Interestingly, we are back to the starting point which hints us that the problem tends to have an iterative solution. In [12], we developed an iterative algorithm for estimating the capacity of finite-buffer line networks. Likewise, here, we present the iterative estimation algorithm depicted in Fig. 4 to estimate the unknown parameters above, starting from arbitrary initial values, *e.g.*, $p_f = 0$. The iteration procedure will go on until convergence of the steady-state probability vectors. One way to measure the convergence of our method is to compare the Euclidean distance between the vectors of each two consecutive iterations and stop the procedure when the distance becomes smaller than a previously chosen threshold.

E. Delay Analysis

Using the iterative estimation algorithm of Section III-D, the steady-state probabilities for RMC and SMC can be obtained. Next, these results are used to find analytical expressions for the average packet latency in DTNs.

We divide the latency experienced by each packet to two parts: "*Network Delay*" and "*Queueing Delay*". The network delay is defined as the total time spent by a packet inside the buffer of a relay node which is the time it takes from the instant when the packet leaves the source node until when it reaches the destination node. The queueing delay is defined as the time spent by a packet inside the queue of the source node which is the time it takes from the instant when the packet arrives at the source node until successfully leaving it. The analytical expressions for both average network delay and average queueing delay at the source are obtained by using the following propositions. The total packet latency can be derived by adding both the network delay and the queueing delay.

Proposition 1 *Let P_z be the portion of the packets that experience zero network delay due to the event that a direct link between the source and the destination is established. Further, let $\Pr\{S_i\}$ is known for $i = 1, 2, \ldots, B$ from the steady-state analysis of RMC. Then, the average network delay is obtained from*

$$D_{net} = \frac{\overline{P_z}}{\overline{p_c}} \sum_{i=1}^{B} \frac{\Pr\{S_i\}}{\sum_{j=1}^{B} \Pr\{S_j\}} \left(E[T_\infty] + (i-1)E[T_0]\right),$$

where the random variables T_0 and T_∞ are defined in Lemma 3, the parameter p_c is derived in Lemma 2 and $P_z = \frac{1}{1 + \left(1-\left(1-\frac{1}{M^2}\right)^n\right)(M^2-1)\ \overline{p_f}}$.

Proposition 2 *Let $\Pr\{A_i\}$ is known for $i = 1, 2, \ldots$ by the steady-state analysis of SMC.Then, the average queueing delay at the source is derived using*

$$D_{queue} = \frac{1}{\overline{p_b}} \sum_{i=1}^{\infty} \frac{\Pr\{A_i\}}{\sum_{j=1}^{\infty} \Pr\{A_j\}} \left(E[T_{\infty,n}] + (i-1)E[T_{0,n}]\right),$$

where the random variables $T_{0,n}$ and $T_{\infty,n}$ are defined in Lemma 5 and the parameter p_b is defined in Lemma 4.

IV. SIMULATION RESULTS

In this section, we present the simulation results for validation of our analytical framework. Our analytical results are compared to simulations of a sparse mobile ad-hoc network with random walk on a grid mobility model.

A mobile network exhibiting random-walk mobility with real time packet transmissions were simulated in MATLAB. The node buffer sizes are chosen to be 10 packets, while the number of relay nodes is kept at 10 and the grid size is 8×8. The mobility parameters needed for Lemmas 1, 4 have not been obtained in closed form in the literature, to the best of our knowledge. However, some approximations are available in [9]. Here, we have obtained the mentioned mobility parameters numerically by a quick simulation of the mobility of two nodes only.

The accuracy of our iterative estimation method is shown in Fig. 5 for average queueing delay at the source and average network delay (in epochs). As stated before, validation of our iterative estimation algorithm is performed for arrival rates λ smaller than the throughput of the network. By increasing

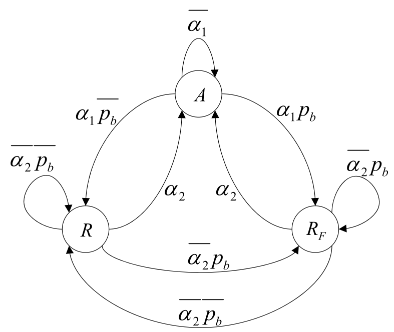

Fig. 6. Three state MC for obtaining α_s and β_s

λ to the values close to the network throughput, the average queueing delay at the source goes to infinity. However, average network delay will remain bounded since all the relays have finite buffer size. In other words, by approaching Λ more and more to the network throughput, queueing delay at the source becomes the dominant term comparing to the network delay.

APPENDIX

TECHNICAL ANALYSIS

A. Sketch of the Proof of Lemma 4

Consider the following subsets of states in the original state-space description of the network.

- A: The most recent event at the source is arrival of a new packet.
- R: The most recent event at the source is meeting a non-full relay or the destination node.
- R_F: The most recent event at the source is meeting a full relay node.

Here, we collapse these subsets into just three states, resulting in the new Markov chain shown in Fig. 6. Clearly, α_s from the original chain in Fig. 3 is given by the probability that the chain in Fig. 6, starting from state A, visits state R before coming back to state A again. Similarly, β_s is given by the probability that the chain in Fig. 6, starting from state R, visits state A before coming back to state R again. Such probabilities can be obtained from the fundamental matrix of the ergodic Markov chain (see chapter 2 of [13] for a discussion on the fundamental Matrix of an ergodic chain) in Fig. 6. Let Z be the fundamental matrix for this chain. The probabilities α_s and β_s can be derived using

$$\alpha_s = \frac{\pi_R}{\pi_A \left\{Z_{RR} - Z_{AR}\right\} + \pi_R \left\{Z_{AA} - Z_{RA}\right\}},$$

$$\beta_s = \frac{\pi_A}{\pi_A \left\{Z_{RR} - Z_{AR}\right\} + \pi_R \left\{Z_{AA} - Z_{RA}\right\}}.$$

The results will follow after performing the necessary computation which would be computing the fundamental matrix Z for Markov chain shown in Fig. 6.

B. Sketch of the Proof of Lemma 5

Considering the network at steady-state, S_0 is the random variable representing the time until the next arrival at the source, given a packet arrival at time $\tau = 0$. At this point, the random location of the other $n+1$ nodes follows the steady-state spatial distribution of the mobility model. Hence, $T_{\infty,n}$ is the random variable representing the waiting time until the source comes in contact with one of the $n+1$ nodes. Further, S_0 and $T_{\infty,n}$ are independent since the arrival process is independent of the mobility. Therefore, the parameter α_1 can be expressed as

$$\begin{aligned}
\alpha_1 &= \Pr\{T_{\infty,n} < S_0\} \\
&= \sum_{\tau=1}^{\infty} \Pr\{S_0 = \tau\} \Pr\{T_{\infty,n} < \tau | S_0 = \tau\} \\
&= \sum_{\tau=1}^{\infty} F_{T_{\infty,n}}(\tau) P_{S_0}(\tau).
\end{aligned}$$

The results for the parameter α_2 can be proved in a similar fashion.

REFERENCES

[1] S. Merugu, M. Ammar, and E. Zegura, "Routing in space and time in networks with predictable mobility," *Technical Report GIT-CC-04-7, Georgia Institute of Technology.*, 2004.

[2] L.-J. Chen, C.-H. Yu, T. Sun, Y.-C. Chen, and H. hua Chu, "A hybrid routing approach for opportunistic networks," *CHANTS '06: Proceedings of the 2006 SIGCOMM workshop on Challenged networks*, pp. 213–220, 2006.

[3] Y. Lin, B. Li, and B. Liang, "Stochastic analysis of network coding in epidemic routing," *IEEE Journal on Selected Areas in Communications*, vol. 26, pp. 794–808, June 2008.

[4] Y. Lin, B. Liang, and B. Li, "Performance modeling of network coding in epidemic routing," in *Proceedings of the 1st international MobiSys workshop on Mobile opportunistic networking*, MobiOpp '07, pp. 67–74, 2007.

[5] R. Groenevelt, P. Nain, and G. Koole, "The message delay in mobile ad hoc networks," *Perform. Eval.*, vol. 62, no. 1-4, pp. 210–228, 2005.

[6] A. Al-Hanbali, A. A. Kherani, and P. Nain, "Simple models for performance evaluation of a class of two-hop relay protocols," *in Proceedings of the IFIP Conference on Networking*, May 2007.

[7] X. Zhang, G. Neglia, J. Kurose, and D. Towsley, "Performance modeling of epidemic routing," *Comp. Net.*, vol. 51, no. 10, pp. 2867–2891, 2007.

[8] T. Spyropoulos, K. Psounis, and C. Raghavendra, "Efficient routing in intermittently connected mobile networks: The single-copy case," *ACM/IEEE Trans. on Networking*, vol. 16, pp. 63–76, February 2008.

[9] R. Subramanian, B. Vellambi, and F. Fekri, "A generalized framework for throughput analysis in sparse mobile networks," in *Proceedings of the 7th International Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks (WiOpt)*, pp. 1–10, June 2009.

[10] R. Subramanian and F. Fekri, "Analysis of multiple-unicast throughput in finite-buffer delay-tolerant networks," in *IEEE International Symposium on Information Theory (ISIT)*, pp. 1634–1638, July 2009.

[11] R. Subramanian and F. Fekri, "Throughput performance of network-coded multicast in an intermittently-connected network," in *Proc. of the 8th International Symposium on Modeling and Optimization in Mobile, Ad Hoc and Wireless Networks (WiOpt)*, pp. 212–221, June 2010.

[12] B. Vellambi, N. Torabkhani, and F. Fekri, "Throughput and latency in finite-buffer line networks," *IEEE Transactions on Information Theory*, vol. 57, pp. 3622–3643, June 2011.

[13] D. Aldous and J. Fill, *Reversible Markov Chains and Random Walks on Graph.* Monograph in preparation, available at http://www.stat.berkeley.edu/users/aldous/RWG/book.html, 2002.