Throughput and Latency of Finite-Buffer Wireless Erasure Networks with Backpressure Routing

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***Abstract*—We consider the problem of estimating throughput and average latency in wireless erasure networks with nodes having finite buffers. In these networks, packets are either lost due to link erasures or dropped because of full buffers. Further, a finite-buffer adaptation of backpressure routing policy is used. The exact Markov chain modeling of such networks for the sake of performance analysis turns out to be an extremely difficult problem in general due to the large number of states and their complicated transitions. In this paper, we propose a novel iterative method that estimates the performance parameters of such networks with much less complexity comparing to the exact analysis. The proposed framework leads to an accurate estimate of the steady-state probability distribution of buffer occupancies using which analytical expressions are obtained for throughput and average packet delay in the network. Finally, these analytical results are validated via simulations.**

I. Introduction

In wireless networks, packets may have to be stored at intermediate nodes to be served at different times due to scheduling and packet loss. In the infinte buffer case, the intermediate nodes do not block (drop) the arriving packets. However, often times, buffers are limited in size and hence, the arriving packets may be dropped due to the congestion of buffers. Although a large buffer size is usually affordable and preferred to minimize packet drops, in many cases, large buffers increases both the average packet delay and its standard deviation dramatically. In Addition, using larger buffer sizes at intermediate nodes would introduce practical problems such as larger on-chip board space and increased memory-access latency.

The problem of computing the capacity of wireless erasure networks has been widely studied [1]–[3]. However, they only considered infinite buffer in their study. The limitations posed by finite memory is first considered in [4] for a simple line network involving a single intermediate node. Inspired by this work, in [5], [6], we investigated bounds for the information-theoretic capacity of general multi-hop wireline networks. Several challenges arise when extending the study from a single intermediate node to a multi-hop line network as we elaborate in [7]. We proposed an approximation method for computing the performance parameters (throughput and latency) of multi-hop line networks in [7]. Further, we extended these results to other communication scenarios, such as, block-based random linear coding for line networks [8], and general wired networks with lossless feedback and random routing [9].

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Since the seminal paper of Tassiulas and Ephremides which proposed a throughput-optimal joint routing/scheduling algorithm [10] (backpresure routing), there has been a great effort to develop throughput-optimal schemes for different networks [11]–[16]. The authors in [11]–[13] investigated optimal scheduling policies for finite-buffer wired and wireless networks and provided lower and upper bound guarantees for the throughput and average queue backlog, respectively. Here, inspired by the routing/scheduling scheme used in [13] for wireless erasure networks and in [12] for the finite-buffer case, we adapt a modified backpressure routing policy for the sake of our analysis.

In this paper, we focus on the problem of performance analysis in wireless erasure networks and investigate the trade-offs between throughput, average packet delay and buffer size when a modified backpressure routing policy is used. Our approach employs a discrete-time model to approximate the buffer occupancy distributions at the intermediate nodes. We then obtain analytical expressions for throughput and average packet delay in terms of the estimated buffer occupancy distributions.

There has been a lot of emphasis on computer simulations and algorithmic approaches to investigate the performance of networks. However, simulations are inadequate for a comprehensive understanding of the interplay of the network parameters. Moreover, simulations may be very cumbersome for large network sizes and for some parameter choices. Very little insight on the optimality of a given protocol can be gained through simulations. Hence, a framework predicting the performance of different protocols for finite-buffer networks is needed. Having such a framework in hand from our previous works, as our contribution in this paper, we will apply it on a modified backpressure routing protocol for finite-buffer wireless erasure networks.

This paper is organized as follows. First, we present a formal definition of the problem and the network model in Section II. Next, we investigate the tools and steps for finite-buffer analysis in Section III. We then obtain expressions for throughput and average delay in Section IV. Finally, Section V presents our analytical results compared to the simulations.

II. Problem Statement and Network Model

Throughout this paper, the network is denoted by an acyclic directed graph $\overrightarrow{G}(V, \overrightarrow{E})$, where packets can be transmitted over a link $\overrightarrow{e} = (u, v)$ only from node u to node v. The system is analyzed using a discrete-time model, where each node can transmit at most a single packet over a link in a time epoch. The links are assumed to be unidirectional, memoryless and lossy, i.e., packets transmitted on a link $\overrightarrow{e} = (u, v) \in \overrightarrow{E}$ are lost randomly with a probability of $\varepsilon_{\overrightarrow{e}} = \varepsilon_{(u,v)}$. Note that the erasures are due to the quality of links (e.g., noise, fading, and interference) and do not represent packet losses due to finite buffers. Moreover, the packet losses on different links are assumed to be independent. Each node has a buffer size of m packets with each packet having a fixed size. Source and destination pairs are assumed to have sufficient memory to store any data packets. The throughput is defined as the transmission rate of information packets (in packets per epoch) between the source and destination. The delay of a packet is also defined as the time taken from the instant when the source starts transmitting a packet to the instant when the destination receives it. Note that the source node has unlimited innovative packets to inject to the network.

In this paper, we adapt the wireless model used in [1], [13]: For any node $v \in V$ with multiple outgoing links, by the broadcast property of the wireless medium, the same packet is sent over all the outgoing links at the same time epoch t (t is an integer). Further, multiple arriving packets for a node $u \in V$ from different incoming links do not interfere and can be stored in a single epoch[1] if there is enough space available in the buffer of node u. In case there is not enough space available in the buffer, some of the arriving packets will be randomly *blocked* by node u. Further, at each epoch, we assume the transmission of a single packet by every node.

Throughout this paper, we employ the following notations. Let $\mathcal{N}^{-}(u)$ denote the set of all the nodes that can receive packets from u. Likewise, $\mathcal{N}^{+}(u)$ is defined as the set of all the nodes that can send packets to u. node s and node d represent the source and destination nodes respectively. Also, for any $x \in [0, 1]$, $\overline{x} \triangleq 1 - x$.

[1]Interferences are avoided in such environments using some form of time, frequency or code division multiple access schemes.

III. Finite-Buffer Analysis of Backpressure Routing

Here, we study the tools and steps that enable our framework for analyzing the performance of backpressure routing for finite-buffer wireless erasure networks.

A. Communication Scheme

Throughout this paper, our goal is to analyze the performance of Diversity Backpressure Routing (DIVBAR) [13]. DIVBAR is generally desirable because of its flexible approach which can dynamically adjust routing decisions in response to the random outcome of the transmissions. In this scheme, every node $u \in V$ transmits a packet in each epoch (blind packet transmissions). After receiving ACK/NACK feedbacks from the various receivers $\mathcal{R} \subset \mathcal{N}^{-}(u)$, node u chooses the receiver node $v \in \mathcal{R}$ with the largest positive differential backlog (*i.e.*, $Q_u(t) - Q_v(t)$) to take the responsibility of forwarding the packet on the path. Here, the backlog parameter $Q_u(t)$ is defined as the current number of packets stored in any node u at the beginning of the time epoch t. Next, node u and all the other receivers delete the packet from their buffers. The algorithm, also breaks ties arbitrarily and retains the packet in u if no receiver has a positive differential backlog. Note that the backlog parameter of each receiver can simply be included in the ACK/NACK signal to be sent back to node u. Note that, the routing scheme is asymptotically throughput optimal meaning that it achieves the wireless min-cut capacity [1] when the buffer sizes are sufficiently large. However, here we aim to study the interplay of throughput and average latency achieved by the backpressure routing in finite-buffer regime.

B. Approximate Markov Chain for an intermediate Node

Thoroughly investigated in [5] for the exact analysis of a finite-buffer line network, as a result of ergodicity of the corresponding Markov Chains (MC), the problem of identifying the throughput is equivalent to the problem of finding the buffer occupancy distribution of the intermediate nodes. Further, due to the exponential growth in the size of the exact Markov chain, exact calculation of the steady-state probability distributions of the buffer occupancies and the network performance is computationally intractable even for networks of reasonable size[2]. Hence, we propose an approximation method that for every node $u \in V$ updates its queue ($Q_u(t)$) considering: 1. The probability of packet arrival at u from the previous-hop neighbors $\mathcal{N}^{+}(u)$, and 2. The effect of blocking imposed by the next-hop neighbors $\mathcal{N}^{-}(u)$. Hence, we will only consider the dependency of the state transition probabilities of the queue for each node u to the state of the queues corresponding to nodes in $\mathcal{N}^{+}(u)$ and $\mathcal{N}^{-}(u)$. Moreover, the main idea of the approximation framework is

[2]For a network of N intermediate nodes, the exact MC has $(m + 1)^N$ states

to divide the multi-dimensional MC with multiple reflections into multiple simple MCs (*i.e.*, Only $Q_u(t)$ for every node $u \in V$) whose steady-state probability distributions can be calculated separately in terms of the steady state probability distributions of the other related MCs. Note that although each MC process is assumed independent of the other MC processes, the interdependency of the states of their queues are captured by the approximation method via their state transition probabilities.

Consider a node $u \in V$ in a network $\overrightarrow{G}(V, \overrightarrow{E})$ with d_i incoming and d_o outgoing edges and a buffer size of m. Let $\mathcal{N}^+(u) \triangleq \{v_1, \ldots, v_{d_i}\}$ and $\mathcal{N}^-(u) \triangleq \{w_1, \ldots, w_{d_o}\}$. By means of our approximation method, the state transition probabilities for MC of any queue $Q_u(t)$ depend on the steady-state distributions of the queues of nodes in $\mathcal{N}^+(u)$ and $\mathcal{N}^-(u)$. Since there is no prior information about the probability distribution of these queues, the proposed estimation algorithm must be performed iteratively. To determine the state transition probabilities for each MC, we need to know the dynamics of arrival and departure of innovative packets to its corresponding queue. As a result of our approximation assumptions, for every queue $Q_u(t)$, we define multiple incomming and outgoing streams of innovative packets which are assumed to be statistically independent. In our model, since we allow the reception of multiple packets in an epoch, the number of arriving streams is the same as the number of incoming links to a node. Also note that, the occupancy of node u ($Q_u(t)$) directly affects the arrival rates, since the probability that node u is selected as the receiver with the largest positive differential backlog is higher when $Q_u(t)$ is smaller. Further, as a result of the broadcast property, only one packet can be conveyed to the set of receivers which implies that there is only one departing stream. In a similar argument, $Q_u(t)$ has a considerable effect on the departure rate since the expected number of receivers with positive differential backlog increases with $Q_u(t)$.

As a result, given $Q_u(t) = n_u$ for an arbitrary node u, we define the set of arrival rates as $\Lambda_u = \{\lambda_1(n_u), \ldots, \lambda_{d_{in}}(n_u)\}$ and the departure rate as $\Omega_u = \{\mu(n_u)\}$. In other words, $\lambda_i(n_u)$ is the probability of an arrival of a packet at node u coming from node v_i (*i.e.*, shifting the responsibility of forwarding a packet from v_i to u) given there are already n_u packets stored at u. Similarly, $\mu(n_u)$ is the probability of a departure of a packet from node u to one of its receivers in $\mathcal{N}^-(u)$. Further, for the systematic representation, we define the arrival and departure polynomials by

$$\begin{aligned} A^{(n_u)}(x) &= \sum_{k=0}^{d_{in}} a_k^{(n_u)} x^k = \prod_{j=1}^{d_{in}} (\overline{\lambda_j(n_u)} + \lambda_j(n_u)x), \\ E^{(n_u)}(x) &= e_0^{(n_u)} + \overline{e_0^{(n_u)}} x = \overline{\mu(n_u)} + \mu(n_u)x. \end{aligned}$$

Let $\Delta_u = \{A^{(n_u)}(x)\}_{n_u=0}^m$ and $\Gamma_u = \{E^{(n_u)}(x)\}_{n_u=0}^m$ be the sets of all arrival and departure polynomials for the queue $Q_u(t)$, respectively. Given Δ_u and Γ_u, the state transition probabilities of the MC for $Q_u(t)$ can be easily computed. As an example, for $0 < j < m$, we have the following[3]:

$$Pr\{Q_u(t+1) = j | Q_u(t) = i\} = \sum_{k=0}^{d_{in}} a_k^{(i)} e_{k+i-j}^{(i)} .$$

Note that the above equation is slightly different for $j = 0$ and $j = m$, whose details are omitted due to the page limit. As a result, the proper approximate MC is formed for $Q_u(t)$ with steady-state probability distribution denoted by $\pi_u(\cdot)$.

In summary, for every node u in network, a queue with its incoming and outgoing streams will be identified properly. Then, the corresponding arrival and departure polynomials will be obtained parametrically. These polynomials describe the state transitions of the queue from which the steady-state probability distribution can be computed for the MC. Then, we apply the following algorithm, denoted as the "Iterative Estimation Algorithm" (IEA), to compute steady-state probability distributions for all the nodes:

1. Initialization (iteration 0): Start with arbitrary rates for the arrival/departure in every node u (*i.e.*, $\Lambda_u^{(0)}$ and $\Omega_u^{(0)}$) and compute $\Delta_u^{(0)}$ and $\Gamma_u^{(0)}$ for each queue, where the superscript denotes the iteration number. However, apply prior information regarding the queues for initialization. For example, in our model, destination node does not block any arriving packet. Also, the source node has infinitely many packets.
2. Increase the iteration index by one (e.g., iteration i). Given $\Delta_u^{(i-1)}$ and $\Gamma_u^{(i-1)}$ for every node u, compute their steady-state probability distributions $\pi_u^{(i-1)}(\cdot)$.
3. Given $\pi_u^{(i-1)}(\cdot)$ for all the MCs, compute the new sets of arrival/departure polynomials $\Delta_u^{(i)}$ and $\Gamma_u^{(i)}$ for every node u.
4. Go back to step 2 until all the steady-state probabilities converge to fixed distributions[4].

Note that at step 3, given the steady state probability distribution for every node, we need to find the arrival/departure polynomials for all the queues. First, we notice that some of the arriving packets on the link (v_i, u) get blocked randomly when all the arriving packets cannot be stored due to node u's state full buffer at time t. Given Δ_u and $Q_u(t) = n_u$, the blocking probability on the link (v_i, u) can be evaluated using

$$p_b\{(v_i,u)|n_u\} = \sum_{H \subset \{0,\ldots,d_i\} \setminus \{i\}} \Big(\prod_{k \in H \setminus \{i\}} \lambda_k(n_u)\Big)\Big(\prod_{k' \in H^c} \overline{\lambda_{k'}(n_u)}\Big) \max\Big\{\tfrac{n_u + |H| + 1 - m}{|H|+1}, 0\Big\} .$$

[3]For notational consistency, we can extend $e_k = 0$ for $k < 0$ or $k > 1$ and $a_k = 0$ for $k < 0$ or $k > d_{in}$.

[4]Convergence of the steady-state probabilities is measured by checking the distance between their estimates for two consecutive iterations and stopping the iterations when the distance becomes less than a certain threshold.

Similarly, a packet is "conveyed" over the link (u, w_i) only when it is not erased on the link and node w_i has the largest positive differential backlog with respect to node u in comparison to all the other successful recipients of the packet at an epoch. Then, given Γ_u and $Q_{w_i}(t) = n_{w_i}$, the rate with which the packets are conveyed over the link (u, w_i) can be obtained as

$$I\{(u,w_i)|n_{w_i}\} = \sum_{l=n_{w_i}+1}^{m} \pi_u(l) \sum_{H\subset\{0,\dots,d_o\}} \left(\prod_{k\in H} \overline{\varepsilon_{(u,w_k)}}\right) \left(\prod_{k'\in H^c} \varepsilon_{(u,w_{k'})}\right) \left(\sum_{Q\subset H} \frac{\prod_{q\in Q\setminus\{i\}} \pi_{w_q}(n_{w_i}) \prod_{q'\in Q^c} \Big(\sum_{j=n_{w_i}+1}^{m} \pi_{w'_q}(j) \Big)}{|Q|} \right).$$

Finally, given $Q_u(t) = n_u$ the arrival/departure rates can be obtained by

$$\begin{aligned}
\mu(n_u) &= 1 - \sum_{Q\subset\{1,\dots,d_{out}\}} \Big(\prod_{k\in Q} \overline{\varepsilon_{(u,w_k)}}\Big) \Big(\prod_{k'\in Q^c} \varepsilon_{(u,w_k)}\Big) \\
&\quad \Big(\prod_{q\in Q} \Big(\sum_{i=0}^{n_u-1} \pi_{w_q}(i) p_b\{(u,w_q)|i\} + \sum_{j=n_u}^{m} \pi_{w_q}(j) \Big)\Big), \\
\lambda_k(n_u) &= I\{(v_k,u)|n_u\}.
\end{aligned}$$

IV. Estimation of the Throughput and Average Packet Delay

In this section, we exploit the results of the iterative estimation method for buffer occupancy distributions in Section III and obtain analytical expressions for throughput and average delay. Since the information rate (The rate of conveying packets) on different links are independent, the throughput estimate $\hat{\mathcal{C}}(s, d, \overrightarrow{G})$ from the source node s to the destination node d is the sum of the information rates arriving to the destination node. Hence,

$$\hat{\mathcal{C}}(s,d,\overrightarrow{G}) = \sum_{v\in\mathcal{N}^+(d)} I\{(v,d)\}.$$

In order to estimate the average delay, one can proceed in a recursive fashion. The average delay that an arriving packet at node $u \in V$ experiences depends on the buffer occupancy of the node u as well as the dynamics of its packet departures. For example, suppose at epoch t (packet arrival time), node u has already n_u packets where $n_u \le m-1$. Then, the arriving packet has to wait for the first n_u packets to leave node u before it can be served. We define $\mathcal{D}_u(n_u)$ as the average time it takes from the instant that node u stores an arriving packet at time t when $Q_u(t) = n_u$ until the time that the destination node receives that packet. Further, Let $\mathcal{L}_u(x, y)$ be the average delay for y packets to depart from node u given it has already x packets in its buffer, $x \ge y$. In order to obtain $\mathcal{D}_u(n_u)$, first we need to compute $\mathcal{L}_u(x, y)$ by solving the corresponding transient MC using the following lemma.

Lemma 1 *Let $T^{(0)}$ and $T^{(1)}$ be $(m+1)\times(m+1)$ matrices, where $T^{(1)}_{i,j}$ is the transition probability from state $\{i-1\}$ to state $\{j-1\}$ for $Q_u(t)$ when a single departure occurs, and $T^{(0)}_{i,j}$ is the transition probability from state $\{i-1\}$ to state $\{j-1\}$ when no departure occurs. Then, given $\mathcal{L}_u(x, 0) = 0$, for $x = 1, \dots, m$ and $y \le x$, we have*

$$\begin{aligned}
\mathcal{L}_u(x,y) = \frac{1}{1-T^{(0)}_{x+1,x+1}} \Big(& 1 + \sum_{i=1}^{m-x} T^{(0)}_{x+1,x+1+i} \mathcal{L}_u(x+i,y) \\
& + \sum_{j=-1}^{m-x} T^{(1)}_{x+1,x+1+j} \mathcal{L}_u(x+j,y-1) \Big).
\end{aligned}$$

Next, using Lemma 1, we compute $\mathcal{D}_u(\cdot)$ for all the intermediate nodes $u \in V$ by the following proposition.

Proposition 1 *Let $\phi_v(n_v)$ for $n_v = 0, 1, \dots, m-1$, be the steady state probability of node $v \in V$ storing n_v packets right before it stores a new arriving packet. In other words, $\phi_v(n_v)$ is the conditional probability of the event $Q_v(t) = n_v$ given that $Q_v(t) < m$. Also, let $I\{(u,v)\} = \sum_{n_v=0}^{m-1} I\{(u,v)|n_v\}\pi_v(n_v)$. Given $\mathcal{D}_v(\cdot)$ for all nodes $v \in \mathcal{N}^-(u)$, for every node $u \in V$, we can obtain $\mathcal{D}_u(n_u)$ for $n_u = 0, 1, \dots, m-1$ using:*

$$\begin{aligned}
\mathcal{D}_u(n_u) = & \quad \mathcal{L}_u(n_u+1, n_u+1) \\
+ & \sum_{w\in\mathcal{N}^-(u)} \sum_{n_w=0}^{m-1} \frac{I\{(u,w)|n_w\}}{\sum_{v\in\mathcal{N}^-(u)} I\{(u,v)\}} \phi_w(n_w)\mathcal{D}_w(n_w) \quad (1)
\end{aligned}$$

Proof: Here, we provide a brief sketch of the proof. Equation (1) is consisted of the following two terms:

1. The first term represents the average time it takes for a total of $k+1$ packets (counting the selected subject packet) to leave node u successfully (and to be stored at one of the next-hop nodes) which is obtained using Lemma 1.
2. The second term relates to the average delay due to the travel of the packet through the rest of the network. The probability of conveying a packet from node u to node w can be estimated by $\frac{I\{(u,w)|n_w\}}{\sum_{v\in\mathcal{N}^-(u)} I\{(u,v)\}}$. An arriving packet at node w finds its buffer already occupied by n_w packets with probability $\phi_w(n_w)$. Thus, the packet will experience an average delay of $\mathcal{D}_w(n_w)$, computed from this node to the destination. Hence, the average packet delay computed from node v to the destination is equal to $\sum_{n_w=0}^{m-1} \frac{I\{(u,w)|n_w\}}{\sum_{v\in\mathcal{N}^-(u)} I\{(u,v)\}} \phi_w(n_w)\mathcal{D}_w(n_w)$.

■

Finally, the total average packet delay, $\mathcal{D}_s(0)$, can be computed by applying Proposition 1 to the source node.

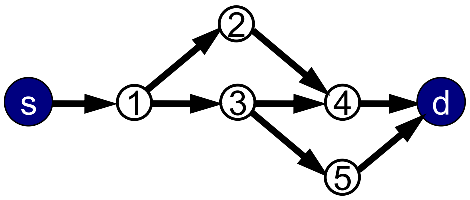

Fig. 1. A sample network.

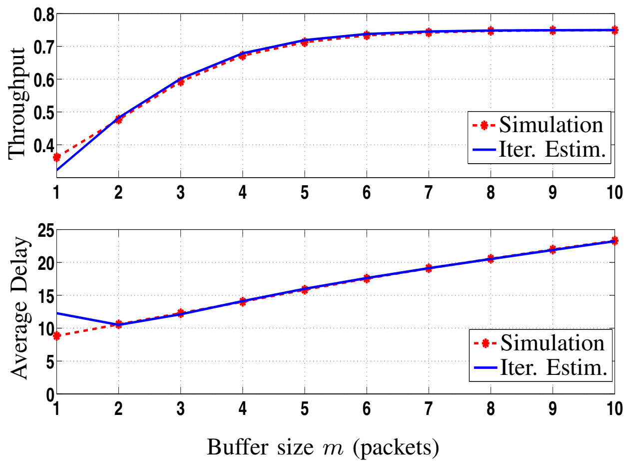

Fig. 2. Throughput and Average packet delay for the sample network.

V. Simulation Results

In this section, we present the results of actual network simulations in comparison with our analysis and show that our framework gives accurate estimates of throughput and average delay.

Consider the sample network in Fig. 1. In this network, all the erasure probabilities are chosen to be 0.5 except for the link $(s, 1)$ which is chosen to be 0.05. Fig. 2 presents the variation of our analytical results and the actual simulations for both throughput and average latency, as the buffer size m is varied. Note that, the throughput is presented in *packets/epoch* and average packet delay is presented in *epochs*. It is noticed that the iterative estimate accurately captures the variation of the performance parameters obtained by simulations. It can be seen that as the buffer size is increased, all curves approach the wireless min-cut capacity [1] of $1 - \varepsilon_{(1,2)}\varepsilon_{(1,3)} = 0.75$. An important observation in this example is that increasing the buffer size beyond $m = 5$ does not improve the throughput significantly. However, it dramatically increases the average latency. This implies that even if a large buffer is available at nodes, it is not a good idea to allocate more than about 5 packets to the same flow. Finally, It can be observed that for $m = 1$ the estimations are not as accurate as the ones for other buffer sizes. The reason could be the separation of dependent MCs as a part of our approximation assumptions mentioned in Section III-B.

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