Delay Analysis of Bursty Traffic in Finite-Buffer Disruption-Tolerant Networks with Two-Hop Routing

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***Abstract*—We consider sparse mobile ad-hoc networks (*i.e.*, disruption-tolerant networks or DTNs) wherein a direct communication path from a source to a destination via multiple hops does not exist due to both mobility and sparseness of the nodes. Hence, the nodes will deliver messages from source to destination using a "store, carry, and forward" strategy. Our goal is to analytically study the packet latency in such networks for a two-hop unicast scenario with bursty packet arrivals at the source. We exploit an embedded Markov chain approach combined with our novel iterative estimation technique to study both network delay and queuing delay. Constraints posed by both the limited node buffer size and contention between nodes for wireless channel are also considered in order to obtain a more realistic model. Finally, our iterative results are validated using simulations for well-known mobility models such as random walk on a grid and the random waypoint mobility.**

I. Introduction

Disruption-tolerant networks (DTNs), also referred to as delay-tolerant networks, are a special type of mobile ad-hoc networks. They are often used when there is no backbone infrastructure and hence have applications in military networks, vehicular networks, and providing basic network services to rural areas.

Conventional mobile ad-hoc Networks (MANETs) rely on the existence of end-to-end paths between source and destination regardless of node mobility. However, simultaneous end-to-end connectivity is very rare in DTNs because of the sparseness of nodes in the network. Hence, communication protocols designed for MANETs are unable to perform efficiently for DTNs. Most of the efficient DTN-based schemes [1], [2], use the "store, carry, and forward" paradigm for message delivery, wherein a source node opportunistically transmits packets upon contacting any other node, and relies on the mobility of these "relay" nodes to deliver the message to a certain destination.

Analytical performance modeling for delay-tolerant networks has recently drawn a considerable amount of attention [3]–[8]. In many cases, the performance of DTNs have been modeled using Poisson process approximations [5]–[7]. Investigated in [9], a major drawback of this approximation is that assuming Poisson process for contact times does not incorporate the spatial-temporal dependence between contact times of any pair of nodes which is not a realistic assumption in general. Inspired by such shortcomings of the previous works, in [9], Subramanian *et al.* proposed a generalized framework for throughput analysis of finite-buffer delay-tolerant networks. The framework uses the embedded Markov chain approach using which the throughput of such networks can be identified by computing certain well-defined characteristic parameters from the mobility model. Further, the problem of throughput analysis in DTNs has been considered for many different communication scenarios and mobility models in [9]–[11], and hence, is well-motivated. Although such a framework is useful and valuable for throughput analysis, it is insufficient for modeling the latency performance of DTNs under different types of source-traffic, for the following reasons:

- In order to compute the throughput in the previous model, the source is assumed to be constantly backlogged, *i.e.*, it has infinite number of information packets. Hence, the relay nodes tend to be as congested as possible. Thus, having such an assumption for the source will lead to computing the maximum average "network delay" only.
- The fact that the source is constantly backlogged will naturally eliminate the necessity of defining queueing delay at the source which is an important performance parameter itself.
- The problem of performance analysis of multiple unicast sessions [10] can be useful only when different sources could have different traffic characteristics. In other words, resource sharing protocols will not have a great impact on the performance of the network if all the flows are backlogged at the source and have the same share from the network resources such as buffer space and bandwidth.

In this paper, as an initial step towards addressing such shortcomings of the previous work [9], we consider the problem of delay analysis for a single unicast session, where a single source node attempts to transmit packets to a single destination using mobile relays. To do so, as our main contribution, a dynamic queue is assumed for the source node with exogenous bursty packet arrivals. By incorporating this seemingly simple addition to the previous problem setting, the new problem turns out to be challenging as we will see in Section III. We will use analytical tools such as embedded Markov chain and Chain-collapsing idea combined with our proposed iterative estimation technique to estimate the steady-state distributions

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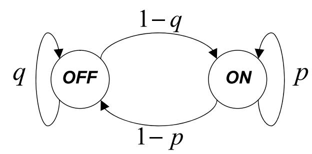

Fig. 1. Bernoulli bursty arrival model

of buffer occupancies for relays and the source. We then use these buffer occupancy distributions to obtain analytical expressions for the average delay of packets in a DTN with a general mobility model. Finally, the analytical results are validated using simulations for certain well-known mobility paradigms such as random walk on a grid and random waypoint mobility.

This paper is organized as follows. First, we present a formal definition of the problem and the network model in Section II. Next, we define the network states and the need for approximation of the exact problem. We investigate the tools and steps for finite-buffer analysis in Section III, and obtain expressions for average network delay and queueing delay in DTNs. Finally, Section IV presents our analytical results compared to the simulations.

II. Network Model

Throughout this paper, the following setup is considered: n identical nodes, referred to as "relay" nodes, and two other nodes, referred to as "source" and "destination" nodes, are located randomly in a field and moving independently according to a certain mobility model. The relay nodes have the same buffer size of B packets where each packet have a fixed length. However, source and destination nodes have unlimited storage capacity. A discrete-time model is used where at each time epoch, only one packet may be transmitted/received by any node. Further, it is assumed that communication is error-free. Analysis of the problem in presence of channel erasure can be shown to be a straightforward extension of the current framework and will not be discussed in this work.

A. Bursty Packet Arrivals at Source

It is known that traffic in communication networks introduces correlations [12], [13]. In this paper, we use a Bernoulli bursty packet arrival process [14] to model such correlations. Packets are generated according to the model depicted in Fig. 1. To be precise, the source alternates between on-periods, during which exactly one packet is generated per time epoch, and off-period, during which no packets are generated. If the source is "On" or "Off", then it remains in the same state with probability p or q, respectively. At each time epoch, the generated packets are stored at source in a buffer with infinite storage capacity, and are served on a first-come first-served basis. Intuitively, it is more convenient to use mean steady-state arrival rate λ (Packets per epoch) and burstiness factor F instead of the parameters p and q. Given λ, the burstiness factor F takes values between $max\{\lambda, 1-\lambda\}$ and infinity and is a measure for the absolute lengths of on/off periods. The burstiness factor of $F = 1$ represents uncorrelated arrivals which is basically a simple Bernoulli arrival model. The parameters λ and F are derived from the following equations [14]

$$\lambda = \frac{1-q}{2-p-q}, \qquad F = \frac{1}{2-p-q}.$$

In this paper, we only consider mean arrival rates λ which are less than the maximum throughput of the network[1], meaning that the queue at the source remains bounded with high probability and hence, the network is stable[2]. This guarantees the boundedness of the average queueing delay at the source. Note that, by choosing λ above the throughput rate, the queue at the source will grow unboundedly since the network could not deliver packets with such a rate. Thus, without loss of generality, in this work, we assume that λ is smaller than the throughput obtained in [9].

B. Interference Model

We assume the communication between a pair of nodes is possible only if they are within the communication range of each other. All the other nodes within the communication ranges of the busy pair are assumed to be silent for the duration of the communication which is one epoch in our problem setup. This is to ensure that there is no wireless interference issues such as hidden-terminal and exposed-terminal situations. Moreover, the source/destination node tries to establish a new link at each epoch, for which several relay nodes may contend. In each time epoch, if the source and destination are within the communication range of each other, then they will form a link, otherwise, if the source or destination are within the communication range of multiple relays, a random relay is selected to setup a link with source or destination, respectively. We say that a "contact" occurs between two nodes whenever they are within the communication range of each other, though they may not communicate. If a pair of nodes win the channel contention, we say that a "link" is established between the communicating nodes.

C. Routing Protocol

Here, we use a two-hop single-copy routing scheme, meaning, whenever a relay node with available space in its buffer establishes a link with the source, it accepts a packet if the source has any packets available in its queue,*i.e.* it is non-empty, and retains the packet until a link is established with the destination. Packets are served on a first-come first-served basis and no relay-to-relay communication occurs. In addition, the source and destination may, though very rarely, establish a direct link.

[1] We define the maximum throughput as the average number of packets delivered to the destination in each time epoch when the network operates at steady state.

[2] Note that, the queues at relays cannot grow to infinity since they have a finite buffer size.

D. Mobility Models

Our framework of analysis is designed to perform well for any mobility model which has stationary properties. This would apply to many well-known models such as random walk on a grid, random waypoint, Brownian motion etc., commonly used in mobile ad-hoc networks research. We assume that each node moves according to the particular chosen mobility model independent of the other nodes in the network. Let $\mathcal{S}_{mob}$ be the set of all states possible in the mobility model. Each "state" of mobility may correspond to information regarding position, direction, velocity, etc. depending upon the underlying mobility model. Let $\chi(t) \in \mathcal{S}_{mob}$ be the state of a single node at any time. It is important to mention that $\chi(t)$ has enough information to determine the probability distribution of $\chi(t+1)$, the state at the next time-step. Typically, one can describe the state transitions for the mobility model by means of a transition function $\boldsymbol{\Psi_{mob}}(\cdot)$ as follows. Let $\mathbf{p}(t)$ be the probability distribution of a node's mobility state at time t. Then, $\mathbf{p}(t+1) = \boldsymbol{\Psi}_{mob}\left[\mathbf{p}(t)\right]$. The transition function $\boldsymbol{\Psi_{mob}}$ depends on the mobility model. Since the mobility model is assumed to be stationary, it has a steady-state probability distribution, $\boldsymbol{\pi}_{mob}$, which satisfies $\boldsymbol{\pi}_{mob} = \boldsymbol{\Psi}_{mob}\left[\boldsymbol{\pi}_{mob}\right]$.

III. Markov Chain Analysis

In a DTN with n relay nodes and a single source destination pair, the *state of the network* is defined as the $(2n+3)$-tuple

$$\mathcal{X}(t) = (\chi_1(t), \cdots, \chi_n(t), \varphi_1(t), \cdots, \varphi_n(t), \chi_s, \varphi_s(t), \chi_d,),$$

where $\chi_k \in \mathcal{S}_{mob}$ is the component describing current mobility state of node k at time epoch t. Also, χ_s and χ_d are the physical mobility states of the source and the destination. The component $\varphi_k(t)$ denotes the buffer occupancy of node k (in packets) at time epoch t. Hence, $0 \leq \varphi_k(t) \leq B$ at any time t for any node k. Also, $\varphi_s(t)$ denotes the buffer occupancy of the source at time epoch t, where $0 \leq \varphi_s(t) < \infty$. Clearly, this describes the state of the network completely: The probabilities of transitions of $\mathcal{X}(t)$ within its state-space can be determined from the mobility model and the communication protocols described previously in Section II. Assuming that the mobility model exhibits stationarity, $\mathcal{X}(t)$ also has a steady state.

The network goes through states wherein packets arrive at the source node, or wherein packets are picked up from the source nodes, or wherein packets are delivered to the destination nodes; these are designated as *active states* for our purpose of delay analysis. Hence, it is sufficient to obtain the steady-state distribution of the entire system described by the state variable $\mathcal{X}(t)$. Steady-state analysis of the network is employed since we are interested in the behavior of the network in the long run. Clearly, the full state-space description of the network is very large to work with. However, we will use the idea of chain-collapsing in the following section to considerably reduce the state-space of the network. Throughout this work, for any $x \in [0,1]$, we define $\overline{x} \triangleq 1 - x$.

A. The Idea of Chain Collapsing

The full state-space description of the network described above is prohibitively large to work with. In order to reduce the state-space and simplify the analysis, we use the idea of chain-collapsing as in [9]. As the first step, we may try to identify certain symmetries in the network that simplifies the state space. For example, in a scenario where relay nodes are identical, one can view the state of the network from a single relay's perspective. However, the state-space is still very large. Note that, by claiming the full state-space description of the network to be very large, we temporarily ignore the state element corresponding to the buffer occupancy of the source $(0 \leq \varphi_s(t) < \infty)$ which is of infinite size. Later, we will observe the challenges of such an extension and will introduce our innovative iterative algorithm to resolve this issue. As the next step, to reduce the state-space further, one can derive a Markov chain from the original state-space such that the steady-state probability distributions are preserved. The above discussion about reducing subsets of states into individual states is thoroughly described in the following theorem from [9]:

Theorem 1 *(Chain Collapsing) Let $\mathcal{M}$ be a Markov chain with a set of states denoted by A, with a steady-state distribution $\boldsymbol{\pi}$ for its states. For each $a \in A$, $\pi(a)$ corresponds to the steady-state probability of state a. Let $\{A_i\}_{i=1}^{z}$ be disjoint subsets of A such that $\bigcup_{i=1}^{z} A_i = A$. Then, a new Markov chain defined with $i = 1 \cdots z$ corresponding to each of the above subsets, with transition probabilities corresponding to the "subset-averaged" values of those from the original Markov chain $\mathcal{M}$, has a steady state distribution $\boldsymbol{\pi'} = [\pi(1)\pi(2)\cdots\pi(z)]$ such that $\pi'(A_i) = \sum_{a_j \in A_i} \pi(a_j)$. Moreover, the transition probabilities for the new chain are given by the following relationship:*

$$p'_{A_r A_l} = \sum_{j \in A_r} \sum_{k \in A_l} p_{jk} \pi(j|A_r) = \frac{1}{\pi'(A_r)} \sum_{j \in A_r} \sum_{k \in A_l} p_{jk} \pi(j)$$

Hence, for a particular relay node, we identify all the "desirable" states which contribute to the time packets spend inside the relays and the source, together with certain additional "auxiliary" states to arrive at an "embedded" Markov chain. The idea of chain-collapsing enables us to extract only the necessary information from the original Markov chain. In particular, the performance computation problem is reduced to computing the steady-state probabilities of certain subsets of a well-defined embedded Markov chain. Note that, we are not interested to find individual steady-state probabilities of states within one particular desired subset. The rest of the analysis involves the computation of the transition probabilities between the desired subsets followed by computation of their steady-state probabilities using the collapsed chain.

B. Embedded Markov Chain for a Relay Node

Here, our main goal is to define the desired states of the embedded Markov chain for a single relay node so that the

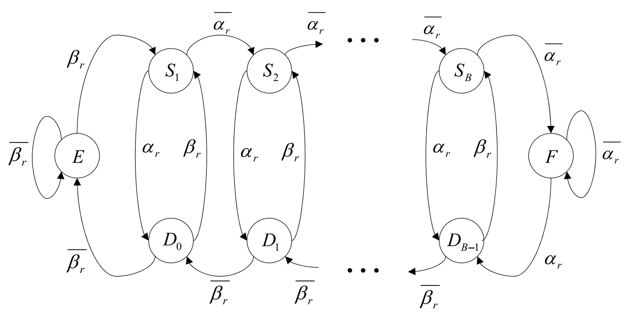

Fig. 2. The embedded Markov chain for a relay node (RMC)

resulting steady-state probabilities could provide us with sufficient information to approach the problem of delay analysis. Hence, we define the embedded Markov chain for a relay node according to the following subsets of states:

- Let S_i ($1 \le i \le B$) be the set of network states wherein the most recent link that node v had was with a non-empty source, resulting in i packets in the buffer after receiving a packet.
- Similarly, let D_j ($0 \le j \le B-1$) be the set of network states wherein the most recent link that node v had was with the destination, resulting in j packets in its buffer after transmitting a packet.
- Let F be the set of network states wherein the most recent link that node v had was with a non-empty source, but v was unable to accept any packet due to lack of buffer space (*i.e.*, Full buffer state).
- Similarly, let E be the set of network states wherein the most recent link that node v had was with the destination, but v had no packet to transmit (*i.e.*, Empty buffer state).

Given the state transition probabilities for the embedded Markov chain in Fig. 2 (RMC), a closed-form expression for its steady-state probabilities can be easily obtained using

$$\begin{aligned}\Pr\{F\} &= \left(\frac{\overline{\alpha_r}}{\overline{\beta_r}}\right)^B \frac{\beta_r}{\alpha_r}\Pr\{E\},\\ \Pr\{S_{i+1}\} = \Pr\{D_i\} &= \left(\frac{\overline{\alpha_r}}{\overline{\beta_r}}\right)^i \frac{\beta_r}{\overline{\beta_r}}\Pr\{E\},\end{aligned}$$

for $i = 0, \dots, B-1$, and,

$$\Pr\{E\} = \begin{cases} \frac{1}{2}\left\{1 + \frac{\beta_r}{\overline{\beta_r}}B\right\}^{-1}, & \text{if } \alpha_r = \beta_r \\ \frac{\alpha_r - \beta_r}{\alpha_r + \beta_r}\left\{1 - \frac{\beta_r}{\alpha_r}\left(\frac{\overline{\alpha_r}}{\overline{\beta_r}}\right)^B\right\}^{-1}, & \text{if } \alpha_r \neq \beta_r \end{cases}.$$

Further, the state transition probabilities for RMC, *i.e.*, α_r and β_r, can be obtained using the following lemma.

Lemma 1 *Let α_0 be the probability that a node currently in contact with the source (or destination) will have a contact with the destination (or source) before coming in contact with the former again. Also, let p_c be the probability that a relay node loses contention on meeting the source/destination node. Finally, let p_e be the probability that source node is empty,* i.e. *has no packets in its queue, when meeting a relay or destination node. Then,*

$$\alpha_r = \frac{\alpha_0}{\overline{p_e}(2\alpha_0 p_c + \overline{p_c}) + \alpha_0 p_e}, \qquad \beta_r = \overline{p_e}\alpha_r.$$

Proof: The proof is very similar to the proof of Lemma 4 (see Appendix). ■

The parameter α_0 in Lemma 1 is characterized for a general mobility model in the following lemma.

Lemma 2 *Let T_0 be a random variable representing the inter-contact duration of two nodes, and let T_∞ be the random variable representing the waiting time until two nodes meet, given that they are distributed according to the steady-state spatial location distribution. Then we have*

$$\alpha_0 = \sum_{\tau=1}^{\infty} F_{T_\infty}(\tau) P_{T_0}(\tau),$$

where $P_{T_0}(\tau)$ and $F_{T_\infty}(\tau)$ are the probability density function of T_0 and the cumulative density function of T_∞, respectively.

Proof: The proof is very similar to the proof of Lemma 6 (see Appendix). ■

Finally, since the contention failure probability p_c in Lemma 1 only depends on the mobility and routing protocol, hence the result from [9] can be exploited to derive p_c using the following lemma.

Lemma 3 *Let $\mathbf{x}'$ be the subset of states wherein a contact with a node in state $\mathbf{x} \in \mathcal{S}_{mob}$ can be established and $\boldsymbol{\pi}_{spt}$ be the steady-state spatial node-location distribution due to the underlying mobility model.*

$$p_c = 1 - \left(1 - E[T_0]^{-1}\right) \sum_{k=0}^{n-1} \sum_{\mathbf{x}} \frac{\pi_{spt}(\mathbf{x})}{k+1} \binom{n-1}{k} \pi_{spt}^k(\mathbf{x}') \times \{1 - \pi_{spt}(\mathbf{x}')\}^{n-1-k}$$

C. Embedded Markov Chain for the Source

Here, we define the desired states of the embedded Markov chain for the source node so that the resulting steady-state probabilities could be helpful for the problem of delay analysis. Hence, the embedded Markov chain for the source node is defined according to the following subsets of states:

- Let A_i ($i = 1, 2, \dots$) be the set of network states wherein the most recent event at the source is a packet arrival (on-period) resulting in i packets in the source queue.
- Also, let R_j ($j = 0, 1, \dots$) be the set of network states wherein the most recent event at the source is meeting a non-full relay or the destination during an off-period, resulting in j packets in the source buffer.
- Finally, let E be the set of network states wherein the most recent event at the source is meeting a non-full relay or the destination during an off-period while the source node is empty and hence no packet is transmitted.

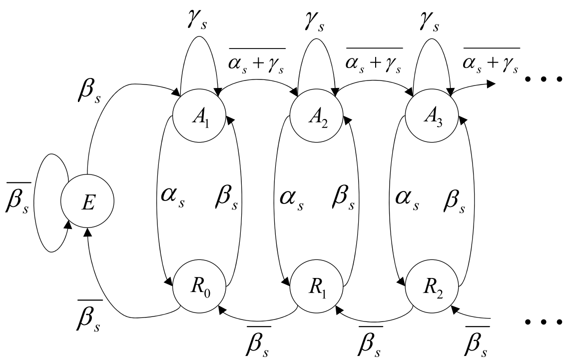

Fig. 3. The embedded Markov chain for the source node (SMC)

Given the state transition probabilities for the embedded Markov chain in Fig. 3 (SMC), a closed-form expression for its steady-state probabilities can be easily obtained using

$$\begin{aligned} \Pr\{R_i\} = \overline{\gamma_s}\Pr\{A_{i+1}\} &= \left(\frac{\overline{\gamma_s + \alpha_s}}{\overline{\beta_s}\ \overline{\gamma_s}}\right)^i \frac{\beta_s}{\overline{\beta_s}} \Pr\{E\}, \quad i \geq 0 \\ \Pr\{E\} &= \frac{\alpha_s - \overline{\gamma_s}\beta_s}{\alpha_s + \beta_s}. \end{aligned}$$

Moreover, the state transition probabilities of SMC, *i.e.*, α_s, β_s and γ_s, can be derived using the following lemmas.

Lemma 4 *Let α_1 be the probability that the source node in its on-period, will have a contact with any other node (relay or destination) before another packet arrives. Further, let α_2 be the probability that the source node, currently in contact with a node (relay or destination), will have an arriving packet before coming in contact with any other node. Finally, let p_f be the probability that a relay node is full when meeting with the source and is unable to accept any packets. Then, we have*

$$\alpha_s = \frac{\alpha_1 \overline{p_b}}{\alpha_2 p_b + \overline{p_b}}, \qquad \beta_s = \frac{\alpha_2}{\alpha_2 p_b + \overline{p_b}},$$

where, $p_b = \frac{n}{n+1} p_f$.

Proof: See Appendix for a brief sketch of the proof. ■

Lemma 5 *Let β_1 be the probability that given the source node has no contacts with any other node (relay or destination), it will contact a node during the next time epoch. Further, let β_2 be the probability that the source node, currently in contact with a node (relay or destination), will have contact with none of the other nodes during the next time epoch. Then, we have*

$$\gamma_s = \frac{\beta_1 \overline{p_b}}{\beta_2 p_b + \overline{p_b}},$$

where, p_b is as defined in Lemma 4.

Proof: The proof is very similar to the proof of Lemma 4 (see Appendix). ■

Finally, the parameters α_1, α_2, β_1 and β_2, can be characterized for a general mobility model and source traffic model using the following lemma.

Lemma 6 *Let S_0 be a random variable representing the inter-arrival duration of packets arriving at source, and let S_∞ be the random variable representing the waiting time until an arrival given the source is currently in off-period. Further, let $T_{\infty,n}$ be a random variable representing the waiting time until a contact with at least one of the $n+1$ relays/destination occur for the source, given that the nodes are distributed according to the steady-state spatial location distribution. Also, let $T_{0,n}$ be a random variable representing the waiting time until source makes contacts with at least one of the $n+1$ relays/destination nodes, given that the source is currently in contact with a relay/destination and the other n nodes are distributed according to the steady-state spatial location distribution. Then, we have*

$$\begin{aligned} \alpha_1 &= \sum_{\tau=1}^{\infty} F_{T_{\infty,n}}(\tau) P_{S_0}(\tau), \\ \alpha_2 &= \sum_{\tau=1}^{\infty} \left(1 - F_{T_{0,n}}(\tau)\right) P_{S_\infty}(\tau), \\ \beta_1 &= \Pr\{T_{\infty,n} = 1\}, \\ \beta_2 &= 1 - \Pr\{T_{0,n} = 1\}. \end{aligned}$$

Proof: See Appendix for a brief sketch of the proof. ■

D. Iterative Estimation

Thus far, we have developed two different collapsed Markov chains RMC and SMC originated from the full state-space of the entire network. In other words, we have observed the desired states in the network from the point of view of a single relay node and the source node. However, it is notable that deriving the state transition probabilities for RMC and SMC requires using Lemmas 1, 4 and 5 in which the parameters p_f and p_e are not known in advance. In this section, we will see that these two Markov chains are not only dependent on each other but also closely related. Further, their dependency could lead us into solving both of them using an iterative algorithm.

We start from the problem of finding the probability p_f. We need to know the portion of relay-source links during which a relay is full. Using steady-state probabilities of RMC, we have the following

$$p_f = \overline{\alpha_0}\ \overline{p_c}\ \frac{\Pr\{F\} + \Pr\{S_B\}}{\Pr\{F\} + \sum_{i=1}^{B} \Pr\{S_i\}}. \tag{1}$$

Further, obtaining the steady-state probabilities of RMC requires having its state transition probabilities by using Lemma 1. Hence, we need to find the probability p_e which is the portion of source-relay/destination links during which the source is empty. Using steady-state probabilities of SMC, the following relation can be obtained

$$p_e = \overline{\alpha_2 + \beta_2}\ \frac{\Pr\{E\} + \Pr\{R_0\}}{\Pr\{E\} + \sum_{i=0}^{\infty} \Pr\{R_i\}}. \tag{2}$$

Finally, obtaining the steady-state probabilities of SMC requires having its state transition probabilities by using Lemmas 4, 5 and consequently, knowing p_f. Interestingly, we are back to where we started. This hints us that the problem might

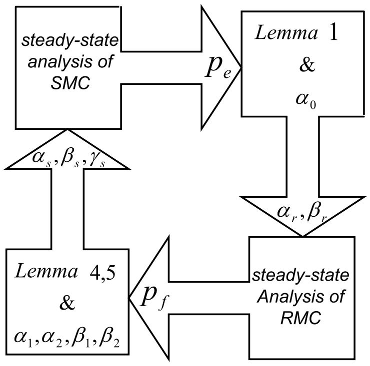

Fig. 4. A graphical presentation of the iterative estimation algorithm

tend to have an iterative solution. In [15], we developed an iterative algorithm to estimate the capacity of finite-buffer line networks (non-mobile). Likewise, here we propose an iterative estimation algorithm to estimate the unknown parameters stated in the discussion above, starting from some initial values, *e.g.*, $p_f = 0$. The schematic in Fig. 4 shows the iteration steps. The iteration procedure will go on until convergence of the steady-state probability vectors. One way to measure the convergence of our method is to compare the Euclidean distance between the vectors of each two consecutive iterations and stop the procedure when the distance becomes smaller than a previously chosen threshold ϵ.

E. Delay Analysis

Using the iterative estimation technique of Section III-D, the estimated steady-state probabilities for RMC and SMC are obtained. In this section, we use such results to find analytical expressions for the average packet delay in DTNs.

We divide the latency experienced by each packet to two parts: "*Network Delay*" and "*Queueing Delay*". The network delay is defined as the total time spent by a packet inside the buffer of a relay node which is the time it takes from the instant when the packet leaves the source node until when it reaches the destination node. The queueing delay is defined as the time spent by a packet inside the queue of the source node which is the time it takes from the instant when the packet arrives at the source node until successfully leaving it. The analytical expressions for both average network delay and average queueing delay at the source are obtained by using the following propositions. The total average packet latency can be simply derived by adding both the average network delay and the average queueing delay.

Proposition 2 *Let P_z be the portion of the packets that experience zero network delay due to the event that a direct link between the source and the destination is established. Given $\Pr\{S_i\}$ for $i = 1, 2, \ldots, B$ from the steady-state analysis of RMC, The average network delay can be obtained from*

$$D_{net} = \overline{P_z} \sum_{i=1}^{B} \frac{\Pr\{S_i\}}{\sum_{j=1}^{B} \Pr\{S_j\}} \left(E[T_\infty] + (i-1)E[T_0]\right),$$

where $P_z = \frac{(n+1)E[T_{0,n}]}{n\overline{p_c}E[T_0]}$, and the contention failure probability p_c is derived in Lemma 3.

Proof: See Appendix for a brief sketch of the proof. ■

Proposition 3 *Given $\Pr\{A_i\}$ for $i = 1, 2, \ldots$ from the steady-state analysis of SMC, the average queueing delay at the source can be derived using*

$$D_{queue} = \overline{p_b}^{-1} \sum_{i=1}^{\infty} \frac{\Pr\{A_i\}}{\sum_{j=1}^{\infty} \Pr\{A_j\}} \left(E[T_{\infty,n}] + (i-1)E[T_{0,n}]\right),$$

where the blocking probability p_b is defined in Lemma 4.

Proof: See Appendix for a brief sketch of the proof. ■

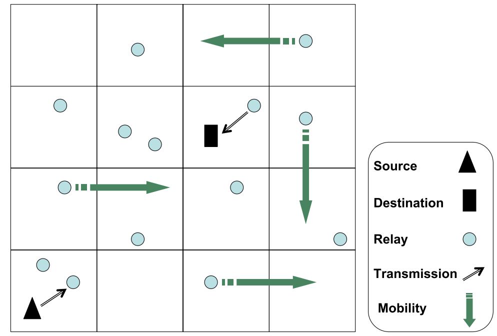

Fig. 5. Network Model

IV. Simulation Results

In this section, we present the simulation results for validation of our analytical framework. Our analytical results are compared to the simulations of sparse mobile ad-hoc networks for two well-known mobility models.

A. Random Walk on a Grid Mobility Model

In this model, nodes are randomly moving on a $M \times M$ square grid as shown in Fig. 5. At each time epoch, nodes may remain in the same cell in the grid, or move to an adjacent cell in the next time step with a certain probability. The transition probabilities for the random walk are chosen so that it results in a uniform steady-state spatial distribution, *i.e.*, a node is located in a specific cell with probability $\frac{1}{M^2}$. Hence, the probability of transition to adjacent cells is $\frac{1}{5}$ and the self-transition probability for each cell will be $1 - \frac{\text{No. of adj. cells}}{5}$. As an example, for the cell in the corner, the self-transition probability is equal to $\frac{3}{5}$. The contention failure probability p_c can be derived using Lemma 3 from the following relation

$$p_c = 1 - \frac{M^2}{n}\left(1 - \left(1 - \frac{1}{M^2}\right)^n\right).$$

The mobility parameters needed for Lemmas 1, 4, 5 can be obtained as well. In [9], an analytical approximation is

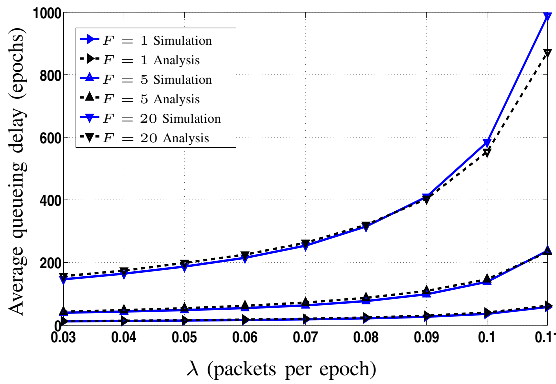

Fig. 6. Variations of average queueing delay at the source with mean arrival rate λ and burstiness factor F for a random walk on a grid mobility model

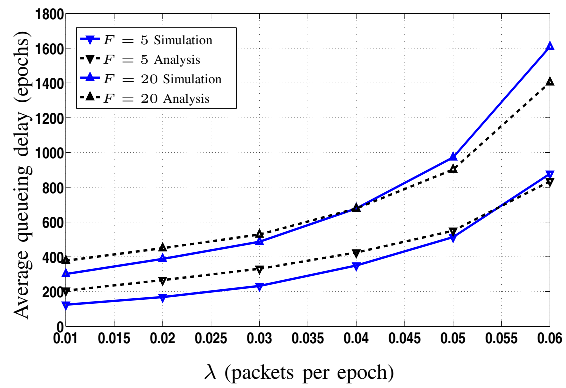

Fig. 8. Variations of average queueing delay at the source with mean arrival rate λ and burstiness factor F for a random waypoint mobility model

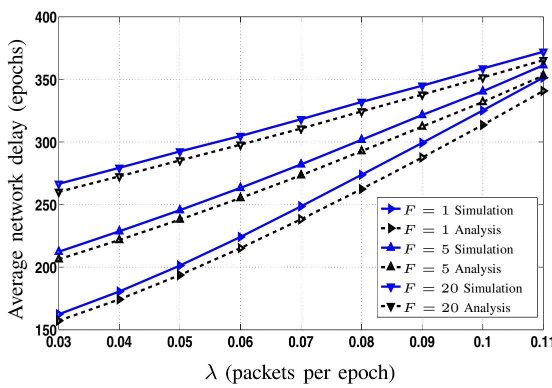

Fig. 7. Variations of average network delay with mean arrival rate λ and burstiness factor F for a random walk on a grid mobility model

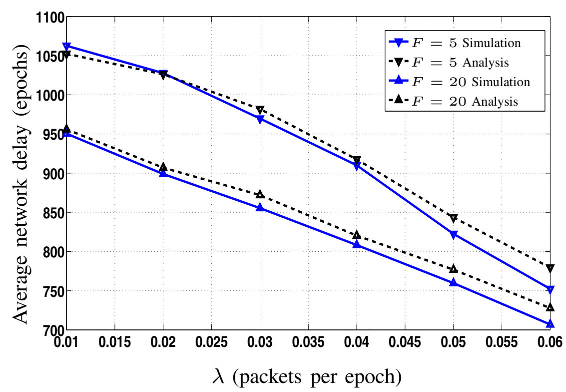

Fig. 9. Variations of average network delay with mean arrival rate λ and burstiness factor F for a random waypoint mobility model

proposed to find such parameters for the case of random-walk on a grid.

Here, we choose the node buffer size to be 10 packets, while the number of relay nodes is kept at 10 and the grid size is 8×8. Fig. 6 and Fig. 7 demonstrate the accuracy of our estimation for average queueing delay at the source and average network delay, respectively. Further, variations of both queueing delay and network delay with mean arrival rate λ and burstiness factor F are presented. As stated before, validation of our iterative estimation algorithm is performed for arrival rates λ smaller than the throughput of the network. By increasing λ to the values close to the network throughput, the average queueing delay at the source goes to infinity. However, average network delay will remain bounded from above since all the relays have finite buffer size. In other words, by approaching more and more to the network throughput, queueing delay at the source becomes the dominant term comparing to the network delay. Finally, it can be observed that, higher burstiness factor results in larger latencies for packets inside the queue of the source.

B. Random Waypoint Mobility Model

The random waypoint mobility model is commonly used in simulation studies of networking protocols. Here, each node selects a random location in the deployment area, and moves towards that location with a random speed. Upon reaching to its target location, it waits for a random amount of time, and then the next location and speed are chosen. The mobility parameters needed for Lemmas 1, 4, 5 have not been obtained in closed form in the literature, to the best of our knowledge. However, some approximations [9] are available for the steady-state spatial node distribution, and can be used to compute the contention failure probability p_c. Here, we have obtained the mentioned mobility parameters numerically by a quick simulation of the mobility only. The deployment area is chosen to be a $5km \times 5km$ square region where 10 nodes are deployed in random locations. The node velocity is chosen from a uniform distribution with $V_{min} = 3m/s$ and $V_{max} = 30m/s$. The communication range is chosen to be $500m$. The pause-time is also modeled as an exponential distribution with a mean of $20s$. Finally, the node buffer size is chosen to be 10 packets.

Fig. 8 and Fig. 9 demonstrate the accuracy of our estimation for average queueing delay at the source and average network delay, respectively. Here, for clarity of the presentation, we only demonstrate the average delay variations for the cases $F = 5$ and $F = 20$ since the curves were close together. It is interesting to observe that, in Fig. 9, by increasing the mean arrival rate λ the average network delay decreases. The reason for such a behavior is solely contributed to the slow nature of the specific mobility parameters (due to lower speed comparing to the area and the waiting periods). As an example, for the random waypoint mobility, the quantity $E[T_{\infty,n}]$ is about 30 times larger than $E[T_{0,n}]$, where the former is only about 2 times larger than the latter for the case of random walk on a grid mobility model. This means that when the mean arrival rate increases, many packets will be trapped inside the relays and take too much time to be released while keeping the relays full. Meanwhile, the proportion of packets with zero network delay will increase as the proportion of the packets transferring directly from source to the destination increases. However, this comes with a cost which would be a huge increase in average queueing delay as it can be observed in Fig. 8.

V. Conclusion

We have considered disruption-tolerant networks (DTNs) wherein a direct path between two particular nodes does not exist due to the mobility and sparseness of the nodes. Hence, the nodes will deliver messages from source to destination using a "store, carry, and forward" strategy. Our goal is to obtain analytical expressions for packet latency in such networks for any mobility model which has stationary properties. Since, the full state-space description of the network is very large, to reduce the state-space and simplify the analysis, we use the idea of chain-collapsing, meaning that, for a particular relay node and the source node, we identify all the "desirable" states which contribute to the delay problem, together with certain additional "auxiliary" states to arrive at an "embedded" Markov chain. Then, we developed two collapsed Markov chains RMC and SMC originated from the full state-space of the network. However, these two Markov chains are not only dependent on each other but also closely related. Further, their dependency leads us into solving both of them using our proposed iterative estimation technique. We have considered constraints posed by limited buffer size as well as contention between nodes for wireless channel to obtain a more realistic model. Finally, our analytical results are validated using simulations for mobility models such as two-dimensional random walk and the random waypoint mobility model.

Appendix

Technical Analysis

A. Sketch of the Proof of Lemma 4

Consider the following subsets of states in the original state-space description of the network.

- A: The source is in its on-period and its most recent event was a packet arrival.
- R: The source is in its off-period and its most recent event was meeting a non-full relay or the destination.
- R_F: The source is in its off-period and its most recent event was meeting a full relay.

Here, we collapse these subsets into just three states, resulting in the new Markov chain shown in Fig. 10. Clearly, α_s from the original chain in Fig. 3 is given by the probability that the chain in Fig. 10, starting from state A, visits state R before coming back to state A again. Similarly, β_s is given by the probability that the chain in Fig. 10, starting from state R, visits state A before coming back to state R again. Such probabilities can be obtained from the fundamental matrix of the Ergodic Markov Chain (see chapter 2 of [16] for a discussion on the fundamental Matrix of an ergodic chain) in Fig. 10. Let Z be the fundamental matrix for this chain. The probabilities α_s and β_s can be derived using

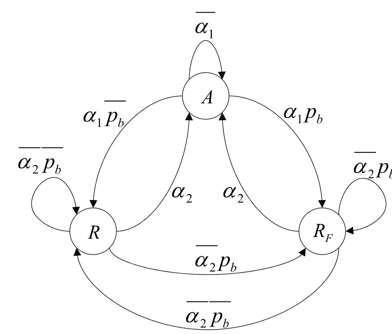

Fig. 10. Three state MC for obtaining α_s and β_s

$$\alpha_s = \frac{\pi_R}{\pi_A\left\{Z_{RR} - Z_{AR}\right\} + \pi_R\left\{Z_{AA} - Z_{RA}\right\}},$$
$$\beta_s = \frac{\pi_A}{\pi_A\left\{Z_{RR} - Z_{AR}\right\} + \pi_R\left\{Z_{AA} - Z_{RA}\right\}}.$$

The results will follow after performing the necessary computation which would be computing the fundamental matrix Z for Markov chain shown in Fig. 10.

B. Sketch of the Proof of Lemma 6

Considering the network at steady-state, S_0 is the random variable representing the time until the next arrival at the source, given that the source is in its on-period at time $\tau = 0$. At this point, the random location of the other $n+1$ nodes follows the steady-state spatial distribution of the mobility model. Hence, $T_{\infty,n}$ is the random variable representing the waiting time until the source comes in contact with one of the $n+1$ nodes. Further, S_0 and $T_{\infty,n}$ are independent since the arrival process is independent of the mobility. Hence, the parameter α_1 can be expressed as

$$\begin{aligned}\alpha_1 &= \Pr\{T_{\infty,n} < S_0\} \\ &= \sum_{\tau=1}^{\infty} \Pr\{S_0 = \tau\} \Pr\{T_{\infty,n} < \tau | S_0 = \tau\} \\ &= \sum_{\tau=1}^{\infty} F_{T_{\infty,n}}(\tau) P_{S_0}(\tau).\end{aligned}$$

The results for the parameter α_2 can be proved in a similar fashion.

As for the parameter β_1, given that the source node has no contacts with any other node (relay or destination) at time $\tau = 0$, meeting a node during the next time epoch means that $T_{\infty,n} = 1$ and hence the result follows. As for the parameter β_2, given that the source node is in contact with a node (relay or destination) at time $\tau = 0$, meeting with none of the other nodes during the next time epoch means that $T_{0,n} > 1$ and hence $\beta_2 = 1 - \Pr\{T_{0,n} = 1\}$.

C. Sketch of the Proof of Proposition 2

Let p_i be the probability that a packet is stored at the i^{th} buffer space of a relay node upon its reception from the source. In this case, such a packet needs to wait for $i-1$ previously stored packets to be delivered to the destination before being served. Hence, for the packet to be delivered to the destination, that particular relay node must establish i links with the destination. Since upon receiving the packet the remaining $n+1$ nodes follow the steady-state spatial distribution of the mobility model, the average waiting time to meet the destination for the first time is $E[T_\infty]$ epochs. Note that meeting the destination node is equivalent to establishing a link with it since there is no contention when the source and the destination are in the same communication range. Afterwards, the remaining $i-1$ links will take an average time of $(i-1)E[T_0]$ epochs to be established. Therefore, by taking an average, we have the following

$$D_{net} = P_z.\,(0) + \overline{P_z}.\left(\sum_{i=1}^{B} p_i \left(E[T_\infty] + (i-1)E[T_0]\right)\right). \quad (3)$$

Further, p_i can be characterized as the conditional probability of having i packets in the buffer of a relay node given that the most recent link that the relay node had is with a non-empty source, and it can be obtained using

$$p_i = \frac{\Pr\{S_i\}}{\sum_{j=1}^{B} \Pr\{S_j\}}, \quad (4)$$

where, $\Pr\{S_i\}$ is known for $i = 1, 2, \ldots, B$ from the steady-state analysis of RMC. Next, P_z is the conditional probability of the source meeting the destination given that its most recent link was established with a non-full relay or the destination. After incorporating the contention between relays, P_z can be determined from

$$P_z = \frac{E[T_0]^{-1}}{\frac{n}{n+1}\overline{p_c}E[T_{0,n}]^{-1}}, \quad (5)$$

where, the contention failure probability p_c is derived in Lemma 3. Finally, by plugging (4) and (5) into (3) the result will follow.

D. Sketch of the Proof of Proposition 3

Let p'_i be the probability that a packet is stored at the i^{th} buffer space of the source node upon its arrival. In this case, such a packet needs to wait for $i-1$ previously stored packets to leave the source before being served. Hence, the source node must establish i links with a non-full relay or the destination. Since upon arrival of the packet the remaining $n+1$ nodes follow the steady-state spatial distribution of the mobility model, the average waiting time to meet a relay or the destination for the first time is $E[T_{\infty,n}]$ epochs. Further, because the relays might be full, the average waiting time to establish a link with a non-full relay or the destination for the first time would be $E[T_{\infty,n}]\overline{p_b}^{-1}$ epochs. Similarly, the remaining $i-1$ links will take an average time of $(i-1)E[T_{0,n}]\overline{p_b}^{-1}$ epochs to be established. Therefore, by taking an average, we have the following

$$D_{queue} = \overline{p_b}^{-1}\sum_{i=1}^{\infty} p'_i \left(E[T_{\infty,n}] + (i-1)E[T_{0,n}]\right), \quad (6)$$

Further, p'_i is the conditional probability of having i packets in the buffer of the source given that the most recent event at the source is a packet arrival, and it can be obtained from

$$p'_i = \frac{\Pr\{A_i\}}{\sum_{j=1}^{\infty} \Pr\{A_j\}}, \quad (7)$$

where, $\Pr\{A_i\}$ is known for $i = 1, 2, \ldots$ from the steady-state analysis of SMC. Finally, by plugging (7) into (6) the result will follow.

References

[1] S. Merugu, M. Ammar, and E. Zegura, "Routing in space and time in networks with predictable mobility," *Technical Report GIT-CC-04-7, Georgia Institute of Technology.*, 2004.

[2] L.-J. Chen, C.-H. Yu, T. Sun, Y.-C. Chen, and H. hua Chu, "A hybrid routing approach for opportunistic networks," *CHANTS '06: Proceedings of the 2006 SIGCOMM workshop on Challenged networks*, pp. 213–220, 2006.

[3] Y. Lin, B. Li, and B. Liang, "Stochastic analysis of network coding in epidemic routing," *IEEE Journal on Selected Areas in Communications*, vol. 26, pp. 794–808, June 2008.

[4] Y. Lin, B. Liang, and B. Li, "Performance modeling of network coding in epidemic routing," in *Proceedings of the 1st international MobiSys workshop on Mobile opportunistic networking*, MobiOpp '07, pp. 67–74, 2007.

[5] R. Groenevelt, P. Nain, and G. Koole, "The message delay in mobile ad hoc networks," *Perform. Eval.*, vol. 62, no. 1-4, pp. 210–228, 2005.

[6] A. Al-Hanbali, A. A. Kherani, and P. Nain, "Simple models for performance evaluation of a class of two-hop relay protocols," *in Proceedings of the IFIP Conference on Networking*, May 2007.

[7] X. Zhang, G. Neglia, J. Kurose, and D. Towsley, "Performance modeling of epidemic routing," *Comp. Net.*, vol. 51, no. 10, pp. 2867–2891, 2007.

[8] T. Spyropoulos, K. Psounis, and C. Raghavendra, "Efficient routing in intermittently connected mobile networks: The single-copy case," *ACM/IEEE Trans. on Networking*, vol. 16, pp. 63–76, February 2008.

[9] R. Subramanian, B. Vellambi, and F. Fekri, "A generalized framework for throughput analysis in sparse mobile networks," in *Proceedings of the 7th International Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks (WiOpt)*, pp. 1–10, June 2009.

[10] R. Subramanian and F. Fekri, "Analysis of multiple-unicast throughput in finite-buffer delay-tolerant networks," in *IEEE International Symposium on Information Theory (ISIT)*, pp. 1634–1638, July 2009.

[11] R. Subramanian and F. Fekri, "Throughput performance of network-coded multicast in an intermittently-connected network," in *Proc. of the 8th International Symposium on Modeling and Optimization in Mobile, Ad Hoc and Wireless Networks (WiOpt)*, pp. 212–221, June 2010.

[12] W. E. Leland, M. S. Taqqu, W. Willinger, and D. V. Wilson, "On the self-similar nature of ethernet traffic (extended version)," *IEEE/ACM Trans. Netw.*, vol. 2, pp. 1–15, February 1994.

[13] V. Paxson and S. Floyd, "Wide area traffic: the failure of poisson modeling," *IEEE/ACM Transactions on Networking*, vol. 3, pp. 226–244, June 1995.

[14] W.-H. Zhou and A.-H. Wang, "Discrete-time queue with bernoulli bursty source arrival and generally distributed service times," *Applied Mathematical Modelling*, vol. 32, no. 11, pp. 2233–2240, 2008.

[15] B. Vellambi, N. Torabkhani, and F. Fekri, "Throughput and latency in finite-buffer line networks," *IEEE Transactions on Information Theory*, vol. 57, pp. 3622–3643, June 2011.

[16] D. Aldous and J. Fill, *Reversible Markov Chains and Random Walks on Graph.* Monograph in preparation, available at http://www.stat.berkeley.edu/users/aldous/RWG/book.html, 2002.