# PAPER Analysis of Block Delivery Delay in Network Coding-Based Delay Tolerant Networks

Juhua PU<sup>†a)</sup>, *Member*, Xingwu LIU<sup>††</sup>, Nima TORABKHANI<sup>†††</sup>, Faramarz FEKRI<sup>†††</sup>, *and* Zhang XIONG<sup>†</sup>, *Nonmembers* 

**SUMMARY** An important factor determining the performance of delay tolerant networks (DTNs) is packet delivery delay. In this paper, we study the block delivery delay of DTN with the epidemic routing scheme based on random linear network coding (RLNC). First, simulations show that the influence of relay buffer size on the delivery delay is not as strong in RLNC-based routing as it is in replica-based routing. With this observation, we can simplify the performance analysis by constraining the buffer of the relay node to just one size. Then we derive the cumulative distribution function (CDF) of block delivery delay with difference equations. Finally, we validate the correctness of our analytical results by simulations.

*key words: delay tolerant network, delay, network coding, epidemic routing, difference equations* 

## 1. Introduction

The delay tolerant network (DTN) is a network architecture that seeks to address the technical issues in heterogeneous networks where continuous network connectivity cannot be guaranteed [1]–[3]. DTNs are usually assumed to consist of sparse nodes moving in a specific area driven by mobility models. Communication can occur only between nodes that happen to be close enough. Thus, there may not be an end-to-end path for a sufficiently long time to fulfill communication from a source to a destination. In DTN, when one node meets another, an opportunistic link can be temporarily established, enabling ad hoc message exchange between them [4]. We say two nodes make contact if they meet with each other and win the channel contention. Thus, the storage-carry-and-forward opportunistic routing protocol is the dominant choice in DTN. That is, a node carries several messages in its buffer and tries to forward them to any node it contacts. Note that a node seldom has a global view of the network status, since this opportunistic strategy makes it impossible for any node to infer a snapshot of the system. Therefore, a big challenge of routing in DTN is how to deliver information efficiently and correctly on the condition that no nodes have a global view of the network status.

<sup>†††</sup>The authors are with the School of Electrical & Computer Engineering, Georgia Institute of Technology, GA, USA.

a) E-mail: pujh@buaa.edu.cn

DOI: 10.1587/transcom.E96.B.135

Thus packet delivery schemes for DTN have been a hot research topic in recent years, among which epidemic routing has been suggested as a viable solution to minimizing latency. In epidemic routing (also called the replicabased routing), when a pair of nodes contact, they select one packet from their buffers and deliver it to the other. So multiple identical packets' copies are injected into the network.

In replica-based routing scheme, deciding which packet should be delivered becomes a key issue and will quite influence the performance. Unfortunately this is hard and has no perfect solution yet, since nodes seldom have global view of the network status. To avoid this difficult choice, network coding is supposed to be quite a promising solution (we call this network-coding-based scheme). Network-coding-based scheme in DTN has attracted much attention from the research community in the past years [5], [6]. A simple and popular one is random linear network coding (RLNC). With RLNC, a node can transmit any coded packet when transmission opportunity occurs, since all of them probabilistically contribute equally to the eventual delivery of all packets to the destination. Thus, if a coded packet is lost, it can be compensated by another coded packet, and retransmission of the same coded packet is unnecessary. Evidence shows that RLNC-based routing can sharply shorten packets delivery delay. But, exactly how short is the delivery delay in RLNC-based routing? This remains an open problem.

In this paper, we try to propose an analysis framework on the delivery delay of RLNC-based routing scheme in DTN. Suppose a source has a block of K packets to transmit to a destination, and we care the latency of this block delivery. In other words, we are interested in the latency with which the destination receives all these K packets. The analysis becomes particularly complicated due to the dependency among packets of relay nodes. For example, two nodes may have each five innovative packets for the destination. However, together, they may have only seven innovative packets for the destination. Hence, if one of the two nodes delivers its five packets to the destination, the other node will have only two innovative packets for the destination.

The remainder of this paper is organized as follows. After surveying the related work in Sect. 2, the network model we focus on is illustrated in Sect. 3. Then, Sect. 4 analyzes some important characteristics of RLNC-based routing scheme in DTN by simulations. Section 5 illustrates a

Manuscript received June 11, 2012.

Manuscript revised August 30, 2012.

<sup>&</sup>lt;sup>†</sup>The authors are with the State Key Laboratory of Software Development Environment, Beihang University, and with Research Institute of Beihang University in Shenzhen, China.

<sup>&</sup>lt;sup>††</sup>The author is with Advanced Computer Research Center, the Institute of Computing Technology, Chinese Academy Science, China.

theoretic analysis framework, followed by the validation in Sect. 6. Finally, we conclude our work in Sect. 7.

## 2. Related Work

To investigate the performance, theoretic analysis is widely used in DTN, in which, such performance metric as delivery delay or throughput are at a premium. [8] analyzes the delay of different redundancy-based routing in DTN by Markov Chain theory. But during the analysis, the authors ignored an important issue of limit storage space, and they didn't concern network coding based routing. Authors in [9] use tandem queue to model a DTN and explore the end-to-end latency theoretically. But this paper only concerns a simple DTN with one fixed source, one fixed destination, and just one mobile relay. Both paper [10] and [11] are focus on the end-to-end delay. [10] aims at the single copy scenario which is seldom used to improve delivery ratio and reduce the delivery delay. Although [11] works for multicopy scenario, its result is mainly for spray-waiting routing strategy while supposing that nodes have infinite buffer size. [12] develops a framework based on Ordinary Differential Equations (ODEs) to study epidemic routing. By this ODE model, the authors present closed-form expressions for such performance metrics as delivery delay, loss probability and power consumption. [13] provides a Markov Chain framework to analyze the throughput of DTN networks, in which bandwidth and limit storage space are taking into account.

Few of aforementioned work are about network coding. paper [14] consider dynamic scheduling for the source to transmit data, and then propose an analytical approach to study the effect of coding on the performance of the network. S. Karande et al. show that network coding does not change the order of throughput in a stationary ad-hoc network [15]. Based on these results, [16] focuses on the performance analysis of network-coded multicast in an intermittently-connected network. Although accurate analysis of network-coded multicast is very complicated, the authors still obtain tight bounds on the performance analysis By deriving a provable way. All these work focus on the analysis of throughput, and seldom is on latency.

In network coding-based routing scheme, since every coded packet can equally contribute to the eventual delivery of all data packets with high probability, the correlation among packets held by different nodes is anfractuous which also makes it much harder to theoretically analyze the performance. To the best of our knowledge, [17] and [18] are the most important results in this field. [17] gives a simulation-based analysis under different scenarios. Lin et al. [18] presents a stochastic analysis of network-codingbased epidemic routing. It has the very similar objective as our work, but is different in two aspects. First, the mobility model in [18] is contact-based where for the sake of more tractability, two consecutive transmission opportunity is assumed to be exponentially distributed. Unfortunately, this exponential distribution model is too ideal to be practical in real world. Second, [18] assumes that every node can get an innovative coded packet from any none-empty nodes unless its buffer is already full of independent packets. This is true with high probability only when the nodes have infinite buffer space, as shown in [19]. However, a finite buffer space is more practical. As a result, we adopt a more realistic mobility model and the nodes are assumed to have finite buffers. The finite buffer assumption entails the consideration of correlations among coded packets.

In summary, the biggest differences between our work and these previous work are that we take into account the finite buffer and the correlations among coded packets, and also our mobility model is more realistic.

## 3. Models and Definitions

#### 3.1 Network and Node Mobility Model

The network scenario we study in this paper is as the follows. There are N + 2 mobile nodes (N relay nodes, and a single pair of source and destination) moving independently in a square area under a grid-based random walk mobility model (referred as the GRW mobility model, hereinafter). We denote these N + 2 nodes as  $V_1, ..., V_{N+2}$ , where  $V_{N+1}$ and  $V_{N+2}$  represent the source and the destination, and  $V_1$  to  $V_N$  are N relays. The source node has a block of K information packets to deliver to the destination. The source and destination nodes have a buffer of size K, while each relay node has a buffer of size B, where  $1 \le B \le K$ . The routing strategy used is RLNC-based epidemic routing, which will be explained in details in Sect. 3.2.

We will assume a discrete-time model for both nodes' mobility and the packet exchanges. That is, at each time step, a node will move to the next location, and also a packet may be exchanged between two nodes which are in the "same" location. The packets delivery delay is denoted by how many steps it will take for the destinations to receive all the K packets from the source. Discrete-time model is a simplification of continuous-time model. This simplification makes it easier to analyze the system, and has been used in much related work [13].

Now we explain the GRW mobility model. We assume that the nodes are deployed on a square region which is divided into  $M \times M$  two-dimensional grids. Each node performs natural random walk on these grids. That is, the motion of a node  $V_i$  is controlled by the following rules: 1)  $V_i$ 's movement is in a discrete time fashion by which  $V_i$  can take at most one movement at each step; 2) in each time step,  $V_i$  remains at the same grid location with probability of  $\frac{1}{5}$  or it moves to one of its four adjacent grid locations with probability of  $\frac{1}{5}$ ; 3) If  $V_i$  reaches the boundary of the square region, it is reflected back and keeps staying in its current grid location. The movement of a node with GRW mobility model is shown in Fig. 1. We assume that the node mobility is at the steady state and wish to analyze the latency of a block delivery from the source to the destination. Obviously, the stationary distribution of the GRW mobility model is a uniform distribution, i.e., each node locates uni-



Fig. 1 Grid-based random walk mobility model.

formly in each cell with probability of  $\frac{1}{M^2}$ .

In this model, two nodes have the opportunity to communicate with each other only when they are in the same cell. But if more than two nodes exist in the same cell, we suppose at most one pair of the nodes randomly chosen from this cell to communicate while others keep silent to avoid contention, and we call these two nodes win the contact.

## 3.2 RLNC-Based Epidemic Routing Scheme

Combining the contention resolution in the above section, our RLNC-based epidemic routing scheme is as follows.

1) The source and destination nodes have higher priority than relay nodes. That is, whenever the source and destination nodes are in the same cell (no matter if there are other relays or not in the same cell), the source can successfully deliver a single coded packet to the destination.

2) If more than one relays meet the source and the destination is not in the same cell, one of the relays is randomly selected and the source tries to deliver one coded packet to this relay. As shown in Fig. 1, in cell (4, 2), there exist the source  $V_{N+1}$  and two relays  $V_1$  and  $V_2$ . Because of the contention, only one of these two relays, say node  $V_1$ , can communicate with and get a packet from  $V_{N+1}$ .

3) If more than one relays meet the destination and the source is not in the same cell, one of the relays is randomly selected and it tries to deliver one coded packet to the destination, if it is not empty. As shown in Fig. 1, in cell (2, 3), there exist the destination  $V_{N+2}$  and three relays  $V_3$ ,  $V_4$ , and  $V_5$ . Only one relay, say node  $V_3$ , can communicate with and deliver a packet to the destination  $V_{N+2}$ .

4) If two or more non-empty relays meet and neither the source nor the destination are in the same cell, two of the relays are randomly selected and they try to exchange one coded packet. As shown in Fig. 1, in cell (3, 3), there exist three relays  $V_6$ ,  $V_7$ , and  $V_8$ .  $V_6$  and  $V_8$  are randomly selected to exchange coded packet between each other.

With RLNC, suppose the source  $V_{N+1}$  tries to deliver a packet to another node  $V_i$ .  $V_{N+1}$  firstly generates a coded packet, i.e., a linear combination of all the *K* source packets from a large Galois field GF(*q*), and then delivers to  $V_i$ this coded packet along with its corresponding coefficients which are randomly chosen from GF(*q*). When receiving a coded packet (from the source or another relay),  $V_i$  will encode this received packet with each content of its buffer as in [16]. In other words, the incoming packet is multiplied to *B* randomly chosen coefficients from the Galois field GF(*q*) and each of the result is added into one of the *B* buffer contents in  $V_i$ . For a relay  $V_i$  to deliver a coded packet (to the destination  $V_{N+2}$  or to another relay),  $V_i$  generates a coded packet by linearly combining all the packets in its buffer. In fact, this coded packets, with each coefficient belonging to the same Galois field GF(*q*). When  $V_i$  delivers this packet, it also sends out these coefficients. Whenever the destination  $V_{N+2}$  receives enough coded packets from the network, it can decode and retrieve the original *K* source packets.

For any node  $V_i$ , we give necessary definitions.

**Definition 1: Coefficient matrix**  $A_i$ . The coefficients of all the packets  $V_i$  holds compose a matrix, we call this matrix as  $V_i$ 's coefficient matrix, denoted by  $A_i$ .

**Definition 2**:  $V_i$ 's Rank  $rank(A_i)$ . The rank of matrix  $A_i$  is called node  $V_i$ 's rank, denoted by  $rank(A_i)$ .  $rank(A_i)$  indicates how many independent packets  $V_i$  holds.

**Definition 3**:  $V_i$ 's **Innovativeness Rank**  $IR_i$ . The number of innovative packets that node  $V_i$  can deliver to the destination  $V_{N+2}$  is called as  $V_i$ 's innovativeness rank W.R.T. the destination (called Innovativeness Rank hereinafter), denoted by  $IR_i$ . It is obvious,  $IR_i = rank \binom{A_i}{A_{N+2}} - rank(A_{N+2})$ .

**Definition 4**:  $V_i$ 's Packet space  $RV_i$ . All the possible linear combinations of the packets in  $V_i$  compose a linear space, referred to as  $V_i$ 's packet space  $RV_i$ . That is, packet space  $RV_i$  is a space spanned by all the packets in  $V_i$ 's buffer.

**Definition 5**: *B*-**buffer-based DTN**. For a DTN, if each of its relays has a buffer of size *B*, we call this DTN *B*-buffer-based DTN. In Sect. 5, the 1-buffer-based DTN means that each relay has a buffer of size one.

#### 4. Simulation-Based Analysis on RLNC-Based DTN

Different from replica-based routing scheme, packets in RLNC-based routing scheme can equally contribute to the eventual delivery of all data packets to the destination with high probability. That is, packets can represent either original packet as needed. With the definition in Sect. 3.2, when  $V_i$  encodes and delivers a packet, it is equivalent to say that  $V_i$  randomly selects and delivers an element from its packet space  $RV_i$ . Thus, for any pair of relays, say  $V_i$  and  $V_j$ , if there exists intersection between their packet spaces  $RV_i$  and  $RV_i$ , e.g., if  $RV_i$  is a sub-space of  $RV_i$ ,  $V_i$ 's innovativeness rank  $IR_i$  might change when  $V_i$  delivers a coded packet to the destination. We look into the example in Sect. 1. Both  $V_i$ and  $V_i$  have each five innovative packets for the destination  $V_{N+2}$ , namely,  $IR_i = 5$  and  $IR_j = 5$ . However, together,  $V_i$ and  $V_i$  have only seven innovative packets for the destination  $V_{N+2}$ . Hence, if  $V_i$  delivers two packets to  $V_{N+2}$ , then,  $IR_i = 3$  and  $IR_i = 5$ . Further, if  $V_i$  delivers a third packet to  $V_{N+2}$ , then,  $IR_i = 3$  and  $IR_i = 4$ . Therefore, the packets' correlations among different nodes may influence the network performance. Altogether with nodes' mobility, it is

IR <sub>j</sub> IR <sub>i</sub>	1	2	3	4	5	6	7	8	9	10
0	0.9991	1	1	1	1	1	1	1	1	1
1	0.4953	0.9990	1	1	1	1	1	1	1	1
2	0.3980	0.6144	0.9975	1	1	1	1	1	1	1
3	0.3644	0.5418	0.6614	0.9988	1	1	1	1	1	1
4	0.3502	0.5252	0.6147	0.7337	0.9991	1	1	1	1	1
5	0.3379	0.5223	0.5983	0.6712	0.7988	0.9989	1	1	1	1
6	0.3221	0.4788	0.5739	0.6401	0.7191	0.7454	0.9990	1	1	1
7	0.3448	0.4890	0.5714	0.5992	0.6601	0.6819	0.7377	0.9992	1	1
8	0.3635	0.4813	0.5344	0.5924	0.6365	0.6643	0.6754	0.7565	0.9990	1
9	0.3677	0.4876	0.5291	0.5908	0.6211	0.6377	0.6319	0.6489	0.6802	0.9951

 Table 1
 Packets correlations between different nodes

even more tough to explore the performance of such kind of DTNs. Thus, we try to achieve some analysis results about the performance via lots of simulations.

#### 4.1 Packets Correlations

In RLNC-based epidemic routing, when a pair of nodes, say  $V_i$  and  $V_i$ , contact, each of them encode and deliver a coded packet to the other. If this coded packet can increase the innovativeness rank of  $V_i$  or  $V_j$ , this contact and transmission conduce to the delivery process of the whole block of K packets (in this case, we call the coded packet is innovative). Thus, we need to care the probability that the coded packet is innovative or not. Recall that, any node  $V_i$ 's innovativeness rank  $IR_i$  is different from its rank  $rank(A_i)$ . So, when  $V_i$  gets a coded packet from  $V_i$ ,  $rank(A_i)$  may increase by 1 while  $IR_i$  remains unchanged. An example is that this coded packet is independent to all the packets  $V_i$  holds, but is dependent to the packets the destination holds (in this case, we call the coded packet is not innovative to  $V_i$ ). So coded packets held by different nodes have complicated correlations for us to explore.

Thus in many existing work, it is common to assume that when  $V_i$  and  $V_j$  contact,  $V_j$  can always transmit an innovative coded packet to  $V_i$  with high probability  $(1 - \frac{1}{q})$  if  $rank(A_j) \ge 1$  and  $rank(A_i) < K$ , and vice verse. When  $rank(A_j) > rank(A_i)$ , the probability is no less than  $1 - \frac{1}{q^{(j-i)}}$ . But when  $rank(A_j) \le rank(A_i)$ , we assert that this is probably not true in real DTN whose nodes have finite buffer.

To validate our assertion, we setup a scenario as K = 10; N = 200, M = 5; B = 10, and  $q = 2^8$  to do a simulation. For any pair of nodes  $V_i$  and  $V_j$ , when  $V_j$  encodes and sends out a packet to  $V_i$ , what is the probability that this packet is innovative to  $V_i$ . We do 500 simulations to statistically get the probability while  $V_i$  and  $V_j$  have different innovativeness rank. As shown in Table 1, in most cases, the probability is far less than 1, which coincides with our assertion. That is, in real DTN when nodes have finite buffer, the packets correlations among different nodes cannot be negligible. That's why we cannot get perfect result when we apply the method in [18] under our GRW mobility model, which can be seen from Fig. 5 to Fig. 8. In our simulations, when j > i, we just approximate the probability to 1 without the statistics calculation to decrease the computing complexity.



Fig. 2 Block delivery delay under different buffer sizes.

## 4.2 Delivery Delay under Different Buffer Sizes

In RLNC-based epidemic routing, since each coded packet can equally contribute to the eventual delivery of all data packets to the destination with high probability, we can predict that nodes' buffer size has not as much influence to the performance as that in replica-based routing. Lin et al. drew the conclusion that the delivery delay under different buffer sizes is very similar for epidemic routing with network coding [18]. Inspired by this result, we try to explore how delivery delay is influenced by buffer size, although our network model is different from that in [18].

To achieve more reasonable results, we setup many scenarios in which block size K, grid size M, and number of nodes N are set to different values to ensure that nodes' density are different as shown in Fig. 2. Under each scenario, the buffer size increases from 1 to 10. As illustrated in Fig. 2, the delivery delay of the RLNC-based routing scheme is seldom influenced by the buffer size in our network model. It is reasonable since any coded packet probabilistically contributes equally to the eventual delivery of all packets to the destination as we addressed in Sect. 1. This result coincides with that in [18] because of similar reasons, and it is significant to our analysis of block delivery delay. We can analyze the delivery delay when each relay has a buffer of size one, and this analysis result can be used to estimate the delivery delay of other buffer size to a certain extent. More importantly, when the buffer size is one, the packets correlations is supposed to be much easier to explore.

#### 5. Delivery Delay of 1-Buffer-Based DTN

Based on the results gotten in Sect. 4, we try to theoretically analyze the block delivery delay of DTN whose relay nodes have buffers of size one and whose routing scheme is RLNC-based epidemic routing. For a relay  $V_i$ , when it contacts with another relay  $V_j$ ,  $V_i$  encodes a packet just by multiplying a coefficient with the packet it holds and sends to  $V_j$ . When  $V_j$  receives a packet,  $V_j$  just put this packet into its buffer if space is available. Otherwise  $V_j$  encodes this received packet with the packet existing in its buffer, and then stores. In this section, we will deduce the CDF of the block delivery delay in such kind of 1-buffer-based DTNs.

#### 5.1 Network Status

Since relays' buffer size is one, their innovativeness rank cannot exceed 1. We use  $\{X_0[T], X_1[T]\}$  to define the network status, where  $X_0[T]$  and  $X_1[T]$  mean, at the time step T, the number of nodes whose innovativeness rank is 0 or 1 respectively. It is obvious that  $\{X_0[T], X_1[T]\}$  is a Markov chain. We have to find out the non-zero transition rates of this chain before we can analyze system's performance.

Now let us give the rule of how to update  $X_0[T]$  and  $X_1[T]$ . By the network status definition, we classify relays into 2 categories. Each node belonging to  $X_1[T]$  category are called type one node, denoted by  $\Lambda_1$ . Otherwise, it is called empty node, denoted by  $\Lambda_0$ . So  $X_0[T] + X_1[T] = N$ , where *N* is the number of relays. When a type one node sends a packet to the destination, it becomes an empty node. When an empty node receive a packet from the source or other type one relays, it becomes a type one node. These rules are helpful to update  $X_0[T]$  and  $X_1[T]$ . But they are not enough to achieve exact update.

As illustrated in Sect. 4, packets carried by different nodes may have correlations. Even each relay has a buffer only of size one, these correlations still remain. For example, as Fig. 3. shows, three relays  $V_i$ ,  $V_j$ , and  $V_m$  whose innovativeness rank satisfies  $IR_i = 1$ ,  $IR_j = 0$ , and  $IR_m = 0$ . Suppose in four successive steps, these nodes transmit packets as illustrated.  $V_i$  sends a packet to  $V_j$  and  $V_m$  at step 1 and step 2 respectively. Then at step 3,  $V_j$  sends to the destination  $V_{N+2}$  a packet which can contribute to the delivery of original packets. At step 4, when  $V_m$  contacts  $V_{N+2}$ ,  $V_m$  contribute nothing to the delivery of original packets. In fact, in this example, when  $V_j$  sends its packet to  $V_{N+2}$ , all the innovativeness rank of  $V_i$  and  $V_m$  should become zero, since packets in these three nodes are identical. And we call all nodes whose packets are identical **equivalence nodes**.

So, to get more accurate analysis, we need further divide type one nodes into different groups, and all the nodes in the same group are equivalence nodes. We call each of these groups **equivalence group**. Thus, when any type one node sends a packet to the destination, all the nodes in the same equivalence group will become empty nodes and this group is destroyed. So the update of  $X_1[T]$  turns to update



Fig. 3 An example about packets correlations.



Fig. 4 Illustration of equivalence groups.

the group information. Now let us give a rough qualitative analysis of how these groups changing (as shown in Fig. 4).

First, when the source  $V_{N+1}$  contacts another node and sends out a packet, it belongs to the following three cases:

**Case 1**:  $V_{N+1}$  contacts an empty node  $\Lambda_0(V_{N+1} \mapsto \Lambda_0)$ . In this case, a new equivalence group should be created, and this empty node belongs to this new group.

**Case 2**:  $V_{N+1}$  contacts  $V_{N+2}(V_{N+1} \mapsto V_{N+2})$ . In this case, the equivalence groups in the network maintain *if f*  $IR_{N+1} > 1$  (the innovativeness rank of  $V_{N+1}$  is larger than one). Otherwise, there is only one group in the network.

**Case 3**:  $V_{N+1}$  contacts a type one node  $\Lambda_1(V_{N+1} \mapsto \Lambda_1)$ . In this case, a new equivalence group should be created, and this type one node belongs to this new group.

Second, when a type one node contacts another relay or the destination and sends out a packet, or a type one node contacts the source and gets a new packet from the source, it still has three cases:

**Case 4**: When a type one node  $\Lambda_1$  contacts an empty node  $\Lambda_0(\Lambda_1 \leftrightarrow \Lambda_0)$ , this empty node joins the equivalence group that this type one node belongs to.

**Case 5**: When a type one node  $V_1$  contacts another type one node  $V'_1(V_1 \leftrightarrow V'_1)$ . If  $V_1$  and  $V'_1$  are from two different equivalence groups, two new equivalence groups should be create.  $V_1$  belongs to one of these two new groups and  $V'_1$ belongs to another. Otherwise, there's no change.

**Case 6**: When a type one node  $\Lambda_1$  contacts the destination  $V_{N+2}$ , the equivalence group this type one node belongs to should be destroyed.

#### 5.2 CDF of Block Delivery Delay

#### 5.2.1 Contact Probability

We say two nodes are in contact if they meet each other

and win the competition to communicate. In our scenario, packet delivery incurs under four types of contacts: 1) The source contacts with the destination with probability  $I_{sd}$ ; 2) A relay contacts with the source with a probability  $I_{rs}$ ; 3) A relay contacts with the destination with a probability  $I_{rd}$ ; 4) A relay contacts with another relay with a probability  $I_{rr}$ . These four probabilities may be different under different mobility models. We can derive their expressions in our GRW model as follows.

$$I_{rs} = \frac{M^2 - 1}{M^4} \sum_{j=0}^{N-1} f\left(j, N - 1, \frac{1}{M^2}\right) \binom{j+2}{2} \tag{1}$$

$$I_{rr} = \frac{(M^2 - 1)^2}{M^6} \sum_{j=0}^{N-2} f\left(j, N - 2, \frac{1}{M^2}\right) \binom{j+2}{2}$$
(2)

where the function  $f(x, y, z) = {y \choose x} z^x (1 - z)^{y-x}$ . Further, we have

$$I_{rs} = I_{rd} \tag{3}$$

$$I_{sd} = \frac{1}{M^2} \tag{4}$$

## 5.2.2 Iteration of $X_1$

As we illustrate in Sect. 5.1, type one nodes are further divided into many equivalence groups, each of which has one or more type one nodes. Suppose at time step *T*, there are totally m(T) equivalence group. For the  $l^{th}$  group, we suppose its group size (number of nodes in this group) is  $a_l(n)$ , and the number of groups with size  $a_l(T)$  is  $b_l(T)$  ( $r \le m(T)$ ). Then, the iteration of these parameters are as follows.

$$a_{l}[T] = \begin{cases} a_{l}[T-1] + I_{rr}a_{l}[T-1]X_{0}[T-1] \\ -\frac{1}{2}I_{rr}a_{l}[T-1](X_{1}[T-1] - a_{l}[T-1]) \\ -I_{rd}a_{l}[T-1] - I_{rs}a_{l}[T-1] (when \ l < m[T]) \end{cases}$$
(5)  
1 (when  $l = m[T]$ )

In above equation,  $I_{rr}a_l[T-1]X_0[T-1]$  shows the case when a type one node contacts an empty node and results in the increase of  $a_l(\operatorname{case} 4)$ ;  $\frac{1}{2}I_{rr}a_l[T-1](X_1[T-1]-a_l[T-1])$ indicates the case when two type one nodes from different groups contact and results in the decrease of  $a_l(\operatorname{case} 5)$ ;  $I_{rd}a_l[T-1]$  and  $I_{rs}a_l[T-1]$  illustrates the case when a type one node contacts the destination results in the decrease of  $a_l(\operatorname{case} 6)$ ;  $I_{rs}a_l[T-1]$  shows the case when a type one node contacts the source and results in the decrease of  $a_l(\operatorname{case} 3)$ .

$$b_{l}[T] = \begin{cases} b_{l}[T-1], (when \ l < m[T]) \\ I_{rs}X_{0}[T-1] + I_{rs}X_{1}[T-1] \\ + \frac{1}{2}I_{rr}\sum_{l=1}^{m[T]} a_{l}[T-1](X_{1}[T-1] - a_{l}[T-1]) \\ (when \ l = m[T]) \end{cases}$$
(6)

In above equations,  $I_{rs}X_0[T-1]$  represents the case when the source contacts an empty node and results in a new group (case 1);  $I_{rs}X_1[T-1]$  illustrates the case when a type one node contacts the source and results in a new group (case 3);  $\frac{1}{2}I_{rr}\sum_{l=1}^{m[T]} a_l[T-1](X_1[T-1]-a_l[T-1])$  shows the case when two type one nodes from different groups contact and results in new groups (case 5).

$$m[T] = m[T - 1] + 1; (7)$$

$$X_{1}[T] = \sum_{l=1}^{m[T]} (a_{l}[T]b_{l}[T])$$
(8)

$$X_0[T] = N - X_1[T]; (9)$$

## 5.2.3 CDF of Block Delivery Delay

Let  $D_j[T]$  denote the probability the destination  $V_{N+2}$  can receive an innovative packet from the network at time step T when its rank is n(n = 0, 1, ...K - 1). Based on the rule for updating  $X_1$ ,  $V_{N+2}$  can get an innovative packet when it contacts any type one node or the source. Thus we have:

$$D_n[T] = \begin{cases} I_{rd}X_1[T] + I_{sd}, 1 < n < K\\ 0, n = K \end{cases}$$
(10)

We further let  $T_n$  denote the deliver delay of n packets, i.e., the time from the beginning to the time when the destination receives n innovative packets from the network. We denote CDF of  $T_n$  by  $P_n(T) = Pr(T_n \le T)$ . Thus,

$$P_{1}(T+1) = P_{1}[T] + D_{0}[T](1 - P_{1}[T])$$
(11)  

$$P_{n}[T+1] = P_{n}[T] + D_{n-1}[T](P_{n-1}[T] - P_{n}[T])$$
(12)  

$$for n = 2, ..., K$$
(12)

The initial value for every  $P_n(T)$  is as  $P_n(0) = 0$ , where n = 1, ..., K. The derivation of the difference equations for  $P_n(T)$  are very similar to that in [19]. So readers can refer to [18] for the detail.

#### 6. Validation

In this section, we compare the derived analysis results with both the empirical results and the analysis results of paper [18]. To compare with the analysis in [18], we also change to adopt the discrete time system by converting the ordinary differential equations to difference equations.

We setup many scenarios in which block size K, grid size M, and number of relay nodes N are set to different values to ensure that nodes' density are different as shown in Fig. 5 to Fig. 8. Each Relay's buffer size is set to one(B =1). In each scenario, we conduct 500 simulations to get the empirical results. In the simulation results shown by Fig. 5 to Fig. 8, the curves denoted by "Contact", "Proposed" and "Simulation" represent the results obtained by the analysis method in [18], or by the analysis method we propose in this paper, or by empirical simulations respectively.



Fig. 5 Simulations (K= 10; N=300; M=10).



Fig. 6 Simulations (K=30; N=150; M=10).



**Fig.7** Simulations (K=10; N=50; M=5).



**Fig. 8** Simulations (K=10; N=150; M=10).

As shown in Fig. 5 to Fig. 8, with relative to the empirical results, our proposed analysis achieves a much closer representation of delay than the contact-based analysis of [18] does. When (K = 10, N = 150, M = 10), both our method and that in [18] have a big gap to the empirical results. But still we can see the distinct gain of our method over that in [18]. We also do more simulations with other parameter settings, and observe significant gains too. We omit these simulations to reduce redundancy.

#### 7. Conclusion

In this paper, we analyze the block delivery delay for DTN with RLNC-based epidemic routing. In this scheme, the correlations among packets carried by different nodes are too complex to explore, and the influence of buffer size on the performance is not as strong as is with replica-based delivery. These factors make it very hard to determine packet delivery delay accurately in RLNC-based delivery scheme which is promising in DTN.

Thus, we first look into the influence of relay buffer size on the performance via simulations which shows that in most cases the buffer size has little influence on the block delivery delay. With this exciting observation, we can concentrate on analyzing the block delivery delay of 1-bufferbased DTN, and its results can be used to approximate other scenarios with large buffer size to some extent. We try to reduce the redundancy among packets by classifying type one nodes into different equivalence groups, and in each group all the nodes carry the "same" packets. Here the "same" means that each pair of packets carried by nodes from the same group are linear dependant and have the same contribution to the delivery of all packets. Then we derive CDF of block delivery delay with difference equations. Simulation results validate the correctness of our analytical results.

#### Acknowledgement

The authors would like to thank the anonymous referees for their valuable comments. This work has been supported by USA National Science Foundation (CCF-0914630), and by China's Natural Science Foundation (61272350 & 61173009), the China's National Programs for High Technology Research and Development (2011AA010502), Doctoral Fund of Ministry of Education of China (20091102110017), and Science Fundation of Shenzhen City in China (JCYJ20120618170520900).

#### References

- I. Psaras, L. Wood, and R. Tafazolli, Delay-/Disruption-Tolerant Networking State of the Art and Future Challenges. Technical report, pp.1–57, 2009. http://www.ee.ucl.ac.uk/~uceeips/dtn-srvipsaras.pdf
- [2] M.J. Khabbaz, C.M. Assi, and W.F. Fawaz, "Disruption-tolerant networking: A comprehensive survey on recent developments and persisting challenges," IEEE Commun. Surveys & Tutorials, vol.14, no.2, pp.607–640, 2012.

- [3] R.D. Souza and J. Jose, "Routing approaches in delay tolerant networks: A survey," Int. J. Computer Applications, vol.1, no.17, pp.8– 14, 2010.
- [4] Q. Zheng, X. Hong, P. Wang, L. Tang, and J. Liu, "Delay management in delay tolerant network," Int. J. Network Management, vol.18, no.6, pp.521–537, 2008.
- [5] K. Chung, Y. Li, and W. Liao, "Exploiting network coding for data forwarding in delay tolerant networks," IEEE 71st Conference on Vehicular Technology, pp.16–19, 2010.
- [6] J. Widmer and J.L. Boudec, "Network coding for efficient communication in extreme networks," SIGCOMM'05, Workshop on Delay Tolerant Networking and Related Networks, pp.1–8, 2005.
- [7] R. Groenevelt, P. Nain, and G. Koole, "The message delay in mobile ad hoc networks," J. Performance Evaluation, vol.62, no.1-4, pp.210–228, 2005.
- [8] Y. Liao, K. Tan, Z. Zhang, and L. Gao, "Modeling redundancy-based routing in delay tolerant networks," 4th IEEE Conference on Consumer Communications and Networking, pp.212–216, 2007.
- [9] A. Hanbali, R. Haan, R. Boucherie, and J. Ommeren, "A tandem queueing model for delay analysis in disconnected ad hoc networks," 15th International Conference on Analytical and Stochastic Modeling, pp.189–205, 2008.
- [10] T. Spyropoulos, K. Psounis, and C. Raghavendra, "Efficient routing in intermittently connected mobile networks: The single-copy case," IEEE Trans. Netw., vol.16, no.1, pp.63–76, 2008.
- [11] T. Spyropoulos, K. Psounis, and C. Raghavendra, "Efficient routing in intermittently connected mobile networks: The multiple-copy case," IEEE Trans. Netw., vol.16, no.1, pp.77–90, 2008.
- [12] X. Zhang, G. Neglia, J. Kurose, and D. Towsley, "Performance modeling of epidemic routing," Elsevier Computer Networks Journal, vol.51, no.10, pp.2859–2891, 2007.
- [13] R. Subramanian, Modeling and Analysis of the Performance of Collaborative Wireless Ad-hoc Networks: An Information-theoretic Perspective, Dissertation for Ph.D., Georgia Institute of Technology, 2009.
- [14] E. Altman, F. De Pellegrini, and L. Sassatelli, "Dynamic control of coding in delay tolerant networks," 2010 Proc. IEEE INFOCOM, pp.1–5, 2010.
- [15] S. Karande, Z. Wang, H. Sadjadpour, and J. Garcia-Luna-Aceves, "Network coding does not change the multicast throughput order of wireless ad hoc networks," Proc. ICC'09, 2009.
- [16] R. Subramanian and F. Fekri, "Throughput performance of networkcoded multicast in an intermittently-connected network," 2010 Proc. 8th International Symposium on Modeling and Optimization in Mobile, Ad Hoc and Wireless Networks (WiOpt), pp.212–221, 2010.
- [17] X. Zhang, G. Neglia, J. Kurose, and D. Towsley, "On the benefits of random linear coding for unicast applications in disruption tolerant networks," Second Workshop on Network Coding, Theory, and Applications, pp.1–7, 2006.
- [18] Y. Lin, B. Li, and B. Liang, "Stochastic analysis of network coding in epidemic routing," IEEE J. Sel. Areas Commun., vol.26, no.5, pp.794–808, 2008.
- [19] S. Deb, M. Medard, and C. Choute, "Algebraic gossip: A network coding approach to optimal multiple ruor mongering," IEEE Trans. Inf. Theory, vol.52, no.6, pp.2486–2507, 2006.



**Juhua Pu** received her B.S. degree from Beijing Jiaotong University in 1999 and the Ph.D. degree from Beihang University (BUAA) in 2005, both in computer science. Now, Dr. Pu is an associate professor in BUAA. She worked as a visiting scholar at the Hongkong University of Science and Technology in 2004, and Georgia Institute of Technology in 2011 respectively. Dr. Pu's current research interests include wireless sensor networks and delay tolerant networks.



Xingwu Liu received his B.S. degree from East China Normal University in 1999, the M.S. degree from Peking University in 2002, both in mathematics, and the Ph.D. degree from the Institute of Computing Technology (ICT), Chinese Academy of Science in 2005. Now, Dr. Liu is an associate professor in ICT. His research interests include distributed computing theory and social networks.



**Nima Torabkhani** received his B.S. degree from University of Tehran in 2007, and the M.S. degree from Georgia Institute of Technology (GIT) in 2010, both in Electrical Engineering. Now Torabkhani is a Ph.D. candidate in GIT, and his research interests include performance analysis of network protocols, network information theory and coding, and delay tolerant networks.







**Zhang Xiong** is currently a professor in the School of Computer Science & Engineer of Beihang University. He received his M.S. degree in computer science from Beihang University in 1984, and visited the Michigan State University from 1989 to 1992. Professor Xiong was awarded China National Golden Medal for Progress in Science and Technology. His current research interests include sensor networks, smart city, etc.