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RESEARCH ARTICLE

# Delay analysis of two-hop network-coded delay-tolerant networks†

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## ABSTRACT

In this paper, we study the block delivery delay of random linear network coding in two-hop single-unicast delay-tolerant networks with grid-based mobility. By block delivery delay, we mean how long it takes the destination to receive all the $K$ information packets of a single block. Our work includes two parts. First, we give a general analysis of the dependency between packet spaces spanned by different nodes in a stochastic way. Then we simplify the result by means of the approximation. By the dependency analysis, we can accurately update nodes' innovativeness rank. Second, via tracking the innovativeness ranks of all nodes, we develop an analytic framework to iteratively compute the cumulative distribution function of the block delivery delay. Our simulation results verify that both parts of our analysis are sufficiently accurate.
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**KEYWORDS**
delay-tolerant networks; network coding; delay analysis

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## 1. INTRODUCTION

The delay or disruption tolerant network (DTN) is a computer network architecture that seeks to address the technical issues in heterogeneous networks where continuous network connectivity cannot be guaranteed [1–3]. DTNs are usually assumed to consist of sparse nodes moving in a specific area driven by mobility models. Communication can occur only between nodes that happen to be close enough. Thus, there may not be an end-to-end path for sufficiently long time to fulfill communicating from a source to a destination. In such networks, when one node meets another, an opportunistic link can be temporarily established, enabling ad hoc message exchange between them. We say two nodes have made contact if they meet with each other and won the channel contention (contact is kind of precious resource in DTN [4]). On this ground, the opportunistic routing protocol has dominantly been used in DTN, which is featured by the store-carry-and-forward strategy. That is, a node carrying packets tries to forward some of the packets to any other in contact. Note that a node seldom has a global view of the network status, as this opportunistic strategy makes it impossible for any node to infer any snapshot of the system. Therefore, one of the challenges of routing in DTN is how to deliver information efficiently and correctly on the condition that none of the nodes have a global view of the network status.

In DTN, nodes' mobility and the packet replication methods influence the performance such as delivery delay and throughput. Suppose a source has a block of $K$ information packets to send to a destination, and we care the latency of this block delivery. In other words, we are interested in the latency with which the destination receives all these $K$ packets. There are two extreme cases. First, if the source meets and transmits a distinct packet to the destination in each of the first $K$ contacts, the delivery delay would be only $K$, reaching the tight lower bound. Second, if the source always transmits a certain packet in any contact, the destination can receive at most one packet, and the delivery delay would tend towards infinity.

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Under certain mobility models, some researchers have focused on designing good packet replication or forwarding strategies to improve delivery performance. To this end, network coding techniques are introduced into DTN [5,6], among which random linear network coding (RLNC) is a simple and popular one. With RLNC, a node can transmit any coded packet, as all of them probabilistically contribute equally to the eventual delivery of all packets to the destination. Thus, if a coded packet is lost, it can be compensated by another coded packet, and hence, retransmission of the same coded packet is unnecessary. Simulation-based results show that RLNC-based routing can significantly shorten packets delivery delay. However, exactly how short is the delivery delay in RLNC-based routing? This remains an open problem.

A plausible way to know the performance of a routing strategy is by simulation. However, this method seems inadequate as enumerating all network circumstances is sometimes infeasible (for example, when the network size is large). Further, it is often difficult to obtain insight to all the trade offs using simulation results. As a result, in this paper, we propose an analytic framework for obtaining the delivery delay of RLNC-based routing for a single block of information packets.

The analysis becomes particularly complicated because of the dependency between packets of relay nodes. For example, two nodes may have each five innovative packets for the destination nodes. However, together, they may have only seven innovative packets for the destination. Hence, if one of the two nodes delivers its five packets to the destination, the other node will have only two innovative packets for the destination.

The remainder of this paper is organized as follows. After a survey of the related work in Section 2, we present the network model in Section 3. Then we explore the dependencies between packets held by different nodes in Section 4. Section 5 elaborates on the analysis, followed by the simulations in Section 6. Section 7 concludes this paper.

## 2. THE RELATED WORK

To investigate the performance, the theoretical analysis is widely used in DTN, which can be divided into two categories: (i) the analysis of mobility models and (ii) the analysis of the routing strategies. Abdulla and Simon [7] showed that inter-contact times of mobile nodes can be closely approximated as exponentially distributed in random waypoint and random direction mobility models. Spyropoulos *et al*. [8] and Chau and Basu [9] predicted the hitting and meeting time of DTN nodes under various mobility models. Bandyopadhyay *et al*. [10] proposed a correlated random walk (CRW) based mobility model and investigates CRW's stochastic properties such as steady-state distribution and state transient metric.

There exist some work on the analysis of the routing strategies [11–13]. Groenevelt *et al*. [14] derived both a closed-form expression and an asymptotic approximation of the expected message delay in mobile ad hoc networks. Liao *et al*. [15] analyzed the delay of different redundancy-based routing in DTN by the Markov chain theory. However, the authors ignored the important issue of the limited storage space. Further, they were not concerned with network coding-based routing. Hanbali *et al*. [16] used tandem queue to model a DTN network and explore the end-to-end latency theoretically. However, the paper is only concerned with a simple DTN with one fixed source, one fixed destination, and a mobile relay. Spyropoulos *et al*. [17,18] are focused on the end-to-end delay of different routing protocols. They [17] aimed at the single-copy scenario, which is seldom used to improve delivery ratio and reduce the delivery delay. Although they [18] worked for multi-copy scenario, their result is mainly for spray-waiting routing strategy while assuming that nodes have infinite-buffer sizes. Subramanian *et al*. [19–21] provided Markov chain frameworks to analyze the throughput of DTN networks, in which bandwidth and the limited storage space are taken into account. Subramanian and Fekri [19] considered throughput and latency of two-hop routing single unicast without network coding, and they extended it to multiple unicast in [20]. They [21] considered throughput for multihop scenario in the steady-state case without network coding (with single-copy routing). None of the aforementioned work studies the transient analysis that arises when a single block delivery is desired. Zhang *et al*. [22] developed a framework based on ordinary differential equations (ODEs) to study epidemic routing. By this ODE model, the authors present closed-form expressions for such performance metrics as delivery delay, loss probability, and power consumption.

Few of aforementioned work are about network coding. In network coding routing, as every coded packet can equally contribute to the eventual delivery of all data packets to the destination with high probability, the dependency between packets held in different nodes' buffers is anfractuous, which also makes it much harder to theoretically analyze the performance. To the best of our knowledge, Zhang *et al*. [23], Subramanian and Fekri [24], and Lin *et al*. [25] are the most important results in this field. Zhang *et al*. [23] gave a simulation-based analysis under different scenarios. Subramanian and Fekri [24] considered throughput of a multihop scenario with network coding. However, this work assumes the steady-state analysis rather than the transient analysis for a single block of information packets. Lin *et al*. [25] presented a stochastic analysis of network-coding-based epidemic routing. It has a similar objective as our work. In [25], the mobility model is contact based where two consecutive transmission opportunities is exponentially distributed with a rate of $\lambda$. This assumption has been made for the sake of more tractability. Further, Lin *et al*. [25] assumed that every node can obtain an innovative coded packet from any nonempty nodes unless its buffer is already full of independent packets. This assumption is made using the analysis result in [26], which has a precondition of infinite-buffer space. Unfortunately, this precondition is not realistic for the finite buffer case. In summary, the analysis in [25] leads to poor estimate of the delay distribution, specially when the mobility does not follow the Poisson distribution and the buffers of the nodes are finite. Finally, Lin *et al*. [25] ignored the dependency of the packets in its analysis.

Inspired by these work, we try to analyze the delivery delay by exploring the correlations among coded packets. That is, how long will it take for the destination to retrieve all the $K$ information packets of a block in a two-hop single-copy network coding routing under the finite-buffer limitation with the grid-based mobility.

# 3. PRELIMINARY

## 3.1. Network model

Our network scenario is as follows. There are $N+2$ mobile nodes ($N$ relay nodes, and a single pair of source and destination) moving independently in a square area under a grid-based random walk mobility model (referred as the GRW model, hereinafter). We denote these $N+2$ nodes as $R_1, \ldots, R_{N+2}$, where $R_{N+1}$ and $R_{N+2}$ represent the source and the destination, and $R_1$ to $R_N$ are $N$ relays. The source node has a block of $K$ information packets to deliver to the destination. The source and destination nodes have a buffer of size $K$, whereas each relay node has a buffer of size $B$, where $1 \leq B \leq K$. The routing strategy used is RLNC-based two-hop single-unicast, which will be explained in details in Section 3.2.

We will assume a discrete-time model for both mobility and the packet exchange. That is, at each time step, a node will move to the next location. Further, at each time step, a packet may be exchanged between two nodes.

Now, we explain the GRW mobility model. We assume that the nodes are deployed on a square region, which is divided into $M \times M$ two-dimensional grids. Each node performs natural random walk on the grids. That is, the motion of a node $R_i$ is controlled by the following rules: (i) $R_i$'s movement is in a discrete-time fashion by which $R_i$ can take at most one movement at each step; (ii) in each time step, $R_i$ remains at the same grid location with probability of $\frac{1}{5}$ or it moves to one of its four adjacent grid locations with probability of $\frac{1}{5}$; (iii) if $R_i$ reaches the boundary of the square region, it is reflected back and keeps staying in its current grid location. The movement of a node with GRW model is shown in Figure 1. We assume that the node mobility is at the steady state and wish to analyze the latency of a block delivery from the source to the destination. As shown in Appendix A, the stationary distribution of the GRW mobility model is a uniform distribution, that is, for any node, it is located in each cell with probability of $\frac{1}{M^2}$.

In the GRW model, a pair of nodes have the opportunity to communicate with each other only when they are in the same grid cell. But if there are more than two nodes in the same grid cell, we suppose at most one pair of the nodes randomly chosen from this grid to communicate while others keep silent to avoid contention.

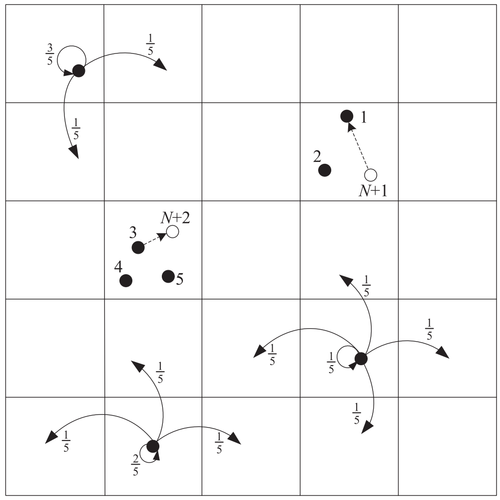

**Figure 1.** Grid-based random walk mobility model.

## 3.2. Packet delivery scheme

Combining the contention resolution in the preceding section, our RLNC-based two-hop single-unicast packet delivery scheme is as follows.

(1) The source and destination have higher priority than relay nodes. That is, whenever the source and the destination are in the same grid cell (no matter if there are other relays or not in the same cell), the source can successfully deliver a single-coded packet to the destination node.
(2) If more than one relay meets the source and the destination is not in the same grid cell, one of the relays is randomly selected, and the source tries to deliver one coded packet to this relay. As shown in Figure 1, in grid cell (4 and 2), there exist the source (labeled as node $N+1$) and two relays (labeled as nodes 1 and 2). Because of the contention, only one of these two relays, say node 1, can communicate with and obtain a packet from the source.
(3) If more than one relay meets the destination and the source is not in the same grid, one of the relays is randomly selected, and it tries to deliver one coded packet to the destination, if it is not empty. As shown in Figure 1, in grid cell (3 and 2), there exist the destination (labeled as node $R_{N+2}$) and three relays, namely nodes $R_3$, $R_4$, and $R_5$. Only one relay, say node $R_3$, can communicate with and deliver a packet to the destination $R_{N+2}$.

With RLNC, suppose the source $R_{N+1}$ tries to deliver a packet to another node $R_i$. $R_{N+1}$ first generates a coded packet, that is, a linear combination of all the $K$ source


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packets from a large Galois field $GF(q)$ and then delivers to $R_i$ this coded packet along with its corresponding coefficients, which are randomly chosen from a Galois field $GF(q)$. When receiving a coded packet, $R_i$ will encode this received packet with each content of its buffer as in [24]. In other words, the incoming packet is multiplied to $B$ randomly chosen coefficients from the field, and each of the result is added into one of the $B$ buffer contents in $R_i$. For a relay node $R_i$ to deliver a coded packet (to the destination $R_{N+2}$), $R_i$ generates a coded packet by linearly combining all the packets in its buffer. In fact, this coded packet is also a linear combination of the original $K$ source packets, with each coefficient belonging to the same Galois field. When $R_i$ delivers this packet, it also sends out these coefficients. Whenever the destination $R_{N+2}$ receives enough coded packets from the network, it can decode and retrieve the original $K$ source packets.

For any node $R_i$, we give the following definitions.

**Definition 1.** *Coefficient matrix $A_i$. The coefficients of all the packets that $R_i$ holds compose a matrix, referred to as $R_i$'s coefficient matrix, denoted by $A_i$.*

**Definition 2.** *$R_i$'s rank $rank(A_i)$. The rank of coefficient matrix $A_i$ is called node $R_i$'s rank, denoted by $rank(A_i)$. The parameter $rank(A_i)$ indicates the number of independent packets that $R_i$ holds.*

**Definition 3.** *$R_i$'s innovativeness rank $IR_i$. The number of innovative packets that node $R_i$ can deliver to the destination $R_{N+2}$ is called $R_i$'s innovativeness rank w.r.t. the destination (called innovativeness rank, hereinafter), denoted by $IR_i$. It is obvious that $IR_i = rank\binom{A_i}{A_{N+2}} - rank(A_{N+2})$.*

**Definition 4.** *Packet space $RV_i$. All the possible linear combinations of the packets in $R_i$ compose a linear space, referred to as $R_i$'s packet space $RV_i$. That is, packet space $RV_i$ is a space spanned by all the packets in $R_i$'s buffer.*

With the preceding definition, when $R_i$ encodes and delivers a packet, it is equivalent to saying that $R_i$ randomly selects and delivers an element from its packet space $RV_i$. Thus, for any pair of relays, say $R_i$ and $R_j$, if there exists intersection between their packet spaces $RV_i$ and $RV_j$, for example, if $RV_i$ is a subspace of $RV_j$, $R_j$'s innovativeness rank $IR_j$ might change when $R_i$ delivers an innovative coded packet to the destination. We look into the example in Section 1. Both $R_i$ and $R_j$ have each five innovative packets for the destination $R_{N+2}$, namely $IR_i = 5$ and $IR_j = 5$. However, together, $R_i$ and $R_j$ have only seven innovative packets for the destination $R_{N+2}$. Hence, if $R_i$ delivers two packets to $R_{N+2}$, then $IR_i = 3$ and $IR_j = 5$. Further, if $R_i$ delivers a third packet to $R_{N+2}$, then $IR_i = 3$ and $IR_j = 4$. Therefore, it is necessary to explore the packets' dependencies among different nodes for an accurate analysis of the network performance.

# 4. DEPENDENCY BETWEEN PACKET SPACES

In this section, we first give a general analysis of the dependency between spaces, calculating the probability that the intersection of two spaces has different dimensions. Then we simplify this approach by some reasonable approximations.

## 4.1. Problem statement

Consider an $n$-dimensional linear space $V$ over Galois field $GF(q)$ ($q$ is the size of the Galois field $GF(q)$). Randomly choose two subspaces $V_s$ and $V_t$ of $V$ with dimensions $s$ and $t$, respectively. We wish to obtain the probability that the intersection of $V_s$ and $V_t$ has dimension $r$ for $s + t - n \le r \le \min\{s, t\}$. That is, we want to calculate $Pr(dim(V_s \bigcap V_t) = r \mid dim(V_s) = s,\ dim(V_t) = t)$. First, we need a few definitions and notations.

Let $N_s$ be the number of $s$-dimensional subspaces of $V$. We define $N^r_{s,t}$ as

$$N^r_{s,t} = |\{(V_s, V_t) : V_s \subseteq V,\ V_t \subseteq V,\ dim(V_s) = s \\ dim(V_t) = t,\ dim(V_s \cap V_t) = r\}| \quad (1)$$

$N^r_{s,t}$ denotes the number of subspace pairs $(V_s, V_t)$, in which $V_s$ and $V_t$ are with dimension $s$ and $t$, respectively, and the subspace intersected by $V_s$ and $V_t$ is with dimension $r$.

Given subspace $V_s \subseteq V$, let

$$\begin{aligned} N^r_{t;V_s} &= |\{V_t : V_t \subseteq V,\ dim(V_t) = t,\ dim(V_t \cap V_s) = r\}| \\ &= |\{(V_s, V_t) : V_t \subseteq V,\ dim(V_t) = t,\ dim(V_t \cap V_s) = r\}| \end{aligned} \quad (2)$$

$N^r_{t;V_s}$ indicates the number of subspaces $V_t$ which is with dimension $t$, and intersects with the given $V_s$ to produce a subspace with dimension $r$.

Given nonnegative integers $j$ and $k$, let $m^k_j = \sum_{i=j}^{k-1} q^i$. If $j \ge k$, $m^k_j = 0$ by convention.

## 4.2. Analysis results

**Lemma 1.** *We have* $Pr(dim(V_s \cap V_t) = r \mid dim(V_s) = s,\ dim(V_t) = t) = \frac{N^r_{t;V_s}}{N_t}$.

*Proof*. Obviously, $N^r_{s,t} = N_s N^r_{t;V_s}$. By definition, $Pr(dim(V_s \cap V_t) = r \mid dim(V_s) = s,\ dim(V_t) = t) = \frac{N^r_{s,t}}{N_s N_t}$. □

**Fact 1.** *Every one-dimensional subspace of $V$ has $q$ elements (hence $q - 1$ nonzero elements). Let $S^n$ stand for the set of all one-dimensional subspaces of $V$. Formally, arbitrarily choose one nonzero element $\alpha_{V_1}$ for each* *one-dimensional subspace $V_1$ of $V$, and hence, $S^n = \{\alpha_{V_1} | V_1 \subseteq V, dim(V_1) = 1\}$. Intuitively, $S^n$ is the projective space of $V$ or, conceivably but not strictly, might be viewed as the unit sphere of $V$.*

**Lemma 2.** *It can be shown that $|S^n| = m_0^n$.*

*Proof*. Obviously, $|V| = q^n$ and $|V \setminus \{0\}| = q^n - 1$. Each one-dimensional subspace has $q-1$ nonzero elements by Fact 1; hence, we have $\frac{q^n-1}{q-1} = m_0^n$. □

**Corollary 1.** *For each $k$-dimensional subspace $V_k$, we have $|V_k \cap S^n| = m_0^k$.*

*Proof*. $|V_k \cap S^n|$ denotes the number of one-dimensional subspaces of $V_k$. Therefore, we have $|S^k| = m_0^k$. □

**Lemma 3.** *Let $V_k$ be a $k$-dimensional linear space over $GF(q)$. There are $\frac{\prod_{i=0}^{j-1} m_i^k}{\prod_{i=0}^{j-1} m_i^j}$ distinct $j$-dimensional subspaces of $V_k$.*

*Proof*. To obtain a $j$-dimensional subspace, we sequentially choose $j$ independent vectors $\alpha_0, \ldots, \alpha_{j-1}$ from $S^k$, which form a basis for the $j$-dimensional subspace. There are $m_0^k$ candidates for $\alpha_0$. As for $\alpha_1$, the only constraint is that it must be different from $\alpha_0$, so there are $m_0^k - 1 = m_0^k - m_0^1 = m_1^k$ candidates for $\alpha_1$. As for $\alpha_2$, the constraint is that it must be out of the subspace $V_2$ spanned by $\alpha_0$ and $\alpha_1$. The intersection of $V_2$ with $S^k$ has $m_0^2$ elements by Corollary 1; thus, there are $m_0^k - m_0^2 = m_2^k$ candidates for $\alpha_2$. Likewise, one can show that for any $l < j$, there are $m_0^k - m_0^l = m_l^k$ candidates for $\alpha_l$. As a result, there are a total of $\prod_{i=0}^{j-1} m_i^k$ distinct bases that can be chosen from $S^k$.

On the other hand, for each $j$-dimensional subspace, there are $\prod_{i=0}^{j-1} m_i^j$ distinct bases that can be chosen from $S^j \subseteq S^k$. The lemma follows. □

**Lemma 4.** *Given $s$-dimensional subspace $V_s$ of $V$, we obtain $N_{t;V_s}^r = \frac{\prod_{i=0}^{r-1} m_i^s \prod_{i=0}^{t-r-1} m_{s+i}^n}{\prod_{i=0}^{r-1} m_i^r \prod_{i=0}^{t-r-1} m_{r+i}^t}$.*

*Proof*. To obtain a $t$-dimensional subspace $V_t$ of $V$ such that $dim(V_t \cap V_s) = r$, one can first choose $r$ independent vectors $\alpha_0, \ldots, \alpha_{r-1}$ from $S^n \cap V_s$ and then choose $t-r$ vectors $\alpha_r, \ldots, \alpha_{t-1}$ from $S^n \setminus V_s$, such that $\alpha_0, \ldots, \alpha_{t-1}$ are independent. Following the proof of Lemma 3, we can choose $\prod_{i=0}^{r-1} m_i^s \prod_{i=0}^{t-r-1} m_{s+i}^n$ different bases each of which spans a $t$-dimensional subspace $V_t$ of $V$ whose intersection with $V_s$ has dimension $r$.

Now, we show how many such bases span the same subspace $V_t$. First, $\alpha_0, \ldots, \alpha_{r-1}$ spans $V_t \cap V_s$ whose dimension is $r$; hence, there are a total of $\prod_{i=0}^{r-1} m_i^r$ candidates for $\alpha_0, \ldots, \alpha_{r-1}$. Second, $\alpha_r$ can be an arbitrary vector from $S^t \setminus (V_t \cap V_s)$; thus, there are $m_r^t$ candidates for it, by the proof of Lemma 3. Following this way, one can show that there are $\prod_{i=0}^{t-r-1} m_{r+i}^t$ candidates for $\alpha_r, \ldots, \alpha_{t-1}$. Altogether, each $V_t$ can be spanned by $\prod_{i=0}^{r-1} m_i^r \prod_{i=0}^{t-r-1} m_{r+i}^t$ different bases that we choose. □

**Theorem 1.** *It can be shown that $Pr(dim(V_s \cap V_t) = r \mid dim(V_s) = s, dim(V_t) = t)$ is given by $\frac{\prod_{i=0}^{r-1} m_i^s \prod_{i=0}^{r-1} m_i^t \prod_{i=s}^{s+t-r-1} m_i^n}{\prod_{i=0}^{r-1} m_i^r \prod_{i=0}^{t-1} m_i^n}$.*

*Proof*. By Lemma 3, $N_t = \frac{\prod_{i=0}^{t-1} m_i^n}{\prod_{i=0}^{t-1} m_i^t}$. By Lemma 4, for an arbitrary $s$-dimensional subspace $V_s$ of $V$, $N_{t;V_s}^r = \frac{\prod_{i=0}^{r-1} m_i^s \prod_{i=0}^{t-r-1} m_{s+i}^n}{\prod_{i=0}^{r-1} m_i^r \prod_{i=0}^{t-r-1} m_{r+i}^t}$.

Hence, by Lemma 1, we obtain

$$
\begin{aligned}
&Pr(dim(V_s \cap V_t) = r \mid dim(V_s) = s,\ dim(V_t) = t) \\
&= \frac{N_{t;V_s}^r}{N_t} \\
&= \frac{\dfrac{\prod_{i=0}^{r-1} m_i^s \prod_{i=0}^{t-r-1} m_{s+i}^n}{\prod_{i=0}^{r-1} m_i^r \prod_{i=0}^{t-r-1} m_{r+i}^t}}{\dfrac{\prod_{i=0}^{t-1} m_i^n}{\prod_{i=0}^{t-1} m_i^t}} \\
&= \frac{\prod_{i=0}^{r-1} m_i^s \prod_{i=0}^{r-1} m_i^t \prod_{i=s}^{s+t-r-1} m_i^n}{\prod_{i=0}^{r-1} m_i^r \prod_{i=0}^{t-1} m_i^n}
\end{aligned} \tag{3}
$$

□

It is worth noting that the result appears to be a complicated expression. Therefore, we must simplify it by approximation.

### 4.3. Approximation

This section focuses on simplifying the formula in Theorem 1. Let $\mu_s^t(r) = \frac{\prod_{i=0}^{r-1} m_i^s \prod_{i=0}^{r-1} m_i^t \prod_{i=s}^{s+t-r-1} m_i^n}{\prod_{i=0}^{r-1} m_i^r \prod_{i=0}^{t-1} m_i^n}$. Note that only $r \geq \max\{s+t-n, 0\}$ makes sense. We try to show that $\mu_s^t(r) \approx 0$ when both $q$ is large enough and $r > \max\{s+t-n, 0\}$. Hence, hereunder, we assume that $r > \max\{s+t-n, 0\}$. Without loss of generality, assume that $s > t$.

**Lemma 5.** *For any $0 \leq i < k \leq n$, it holds that $m_i^k = \frac{q^{k-i}-1}{q-1} q^i$.*

The proof is straightforward and is hence omitted.

**Lemma 6.** *For any $0 \leq j \leq l \leq k \leq n$, we have*

$$
\prod_{i=j}^{l-1} m_i^k = \frac{\prod_{i=j}^{l-1} (q^{k-i} - 1)}{(q-1)^{l-j}} q^{\frac{(l-j)(l+j-1)}{2}}.
$$


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The proof follows immediately from Lemma 5 and is also omitted. As a result, we obtain

$$\mu_s^t(r) = q^{\frac{1}{2}(r(r-1)+r(r-1)+(t-r)(2s+t-r-1)-r(r-1)-t(t-1))} \frac{\prod_{i=0}^{r-1}(q^{s-i}-1)\prod_{i=0}^{r-1}(q^{t-i}-1)\prod_{i=s}^{s+t-r-1}(q^{n-i}-1)}{\prod_{i=0}^{r-1}(q^{r-i}-1)\prod_{i=0}^{t-1}(q^{n-i}-1)}$$
$$= q^{(s-r)(t-r)} \frac{\prod_{i=0}^{r-1}(q^{s-i}-1)\prod_{i=0}^{r-1}(q^{t-i}-1)\prod_{i=s}^{s+t-r-1}(q^{n-i}-1)}{\prod_{i=0}^{r-1}(q^{r-i}-1)\prod_{i=0}^{t-1}(q^{n-i}-1)}$$
$$< q^{(s-r)(t-r)} \frac{\prod_{i=0}^{r-1}q^{s-i}\prod_{i=0}^{r-1}q^{t-i}\prod_{i=s}^{s+t-r-1}q^{n-i}}{\prod_{i=0}^{r-1}q^{r-i}\frac{q^{r-i}-1}{q^{r-i}}\prod_{i=0}^{t-1}q^{n-i}\frac{q^{n-i}-1}{q^{n-i}}}$$
$$= q^{-r(n+r-s-t)}\prod_{i=1}^{r}\frac{q^i}{q^i-1}\prod_{i=n-t+1}^{n}\frac{q^i}{q^i-1} \tag{4}$$

**Lemma 7.** *The inequality* $\frac{q^i}{q^i-1} \le \frac{q^{i-1}+1}{q^{i-1}}$ *holds for* $q \ge 2$.

*Proof*. It follows from the inequality that $q^{2i-1} \le q^{2i-1} + q^i - q^{i-1} - 1$. □

**Lemma 8.** *We have* $\prod_{i=1}^{r}\frac{q^i}{q^i-1} \le e^{\frac{q}{q-1}}$.

*Proof*. : Let $\rho = \prod_{i=1}^{r}\frac{q^i}{q^i-1}$. Then by Lemma 7, we can obtain that $\rho \le \prod_{i=1}^{r}\frac{q^{i-1}+1}{q^{i-1}} = \prod_{i=0}^{r-1}\left(1+\frac{1}{q^i}\right)$. According to the inequality $\log(1+x) \le x$ for any $x > 0$, we have

$$\log\rho \le \sum_{i=0}^{r-1}\log\left(1+\frac{1}{q^i}\right) \le \sum_{i=0}^{r-1}\frac{1}{q^i} \le \sum_{i=0}^{\infty}\frac{1}{q^i} = \frac{q}{q-1} \tag{5}$$

Hence, the lemma holds. □

Likewise, we know that $\prod_{i=n-t+1}^{n}\frac{q^i}{q^i-1} \le e^{\frac{q}{q-1}}$.

**Theorem 2.** *For any* $q \ge 16$, *we obtain the inequality* $\mu_s^t(r) \le e^{\frac{q}{q-1}}q^{-1}$.

*Proof*. We do a case study.

**Case 1.** $r \ge 2$. Then by Lemma 8, we can obtain that $\mu_s^t(r) \le q^{-r(n+r-s-t)}\prod_{i=1}^{r}\frac{q^i}{q^i-1}\prod_{i=n-t+1}^{n}\frac{q^i}{q^i-1} \le q^{-2}(e^{\frac{q}{q-1}})^2$. As $q \ge 8$, $e^{\frac{q}{q-1}}q^{-1} < 1$, so $\mu_s^t(r) \le q^{-1}e^{\frac{q}{q-1}}$.

**Case 2.** $r = 1$. As $s \ge k, t \ge k, r > s+t-n$, we have $n \ge 2$. Further, $n-t+1 > 1$, because if it does not hold, then $n \le t \le s$, implying that $s+t-n = n \ge r$, which is contradictory to the assumption that $r > s+t-n$. Hence,

$$\prod_{i=1}^{r}\frac{q^i}{q^i-1}\prod_{i=n-t+1}^{n}\frac{q^i}{q^i-1} = \frac{q}{q-1}\prod_{i=n-t+1}^{n}\frac{q^i}{q^i-1}$$
$$\le \frac{q}{q-1}\prod_{i=2}^{n}\frac{q^i}{q^i-1} = \prod_{i=1}^{n}\frac{q^i}{q^i-1} \tag{6}$$

which is no greater than $e^{\frac{q}{q-1}}$ by Lemma 8. Therefore, on the basis of expressions (4), $\mu_s^t(r) \le q^{-(n+r-s-t)}e^{\frac{q}{q-1}} \le e^{\frac{q}{q-1}}q^{-1}$.

Altogether, the theorem holds. □

## 4.4. Packet dependency

Let $r_0 = s+t-n$. By Theorem 2, if $q$ is large enough (e.g., $q \ge 256$), then $e^{\frac{q}{q-1}} < 3$ and $\mu_s^t(r) \le \frac{3}{q}$. In this case, $\mu_s^t(r \mid r > max\{r_0, 0\})$ is negligible; hence, we only need to be concerned with $\mu_s^t(\max\{r_0, 0\})$.

Recall that we try to explore the dependency between packets spaces of different nodes. If $r_0 \le 0$, this means that there would be no dependency among two nodes' packets spaces. Thus, we only need to consider the following situation: $0 < r_0 \le s$.

**Theorem 3.** *When* $q \ge 16$ *and* $r_0 > 0$, *we have* $Pr(dim(V_s \cap V_t) = r_0 \mid dim(V_s) = s,\ dim(V_t) = t) > 1 - q^{-1}$.

*Proof*. We still assume $s \le t$ without loss of generality. With the definition of $\mu_s^t(r)$, only when $r_0 \le r \le s$, $\mu_s^t(r)$ makes sense. We have $\mu_s^t(r_0) = 1 - \sum_{r=r_0+1}^{s}\mu_s^t(r)$. As $r_0 > 0$, then $r = r_0 + 1 \ge 2$. Consider a case study.

**Case 1.** $r_0 + 1 < s$. That is, $\mu_s^t(r_0) = 1 - \sum_{r=r_0+1}^{s}\mu_s^t(r)$. For each $r_0 + 1 \le r \le s$, we first calculate $\sum_{r=r_0+1}^{s}\mu_s^t(r)$.

By Lemma 8,

$$\sum_{r=r_0+1}^{s} \mu_s^t(r) \le \sum_{r=r_0+1}^{s} q^{-r}\left(e^{\frac{q}{q-1}}\right)^2 = \frac{\left(e^{\frac{q}{q-1}}\right)^2}{q}\frac{q^{-r_0}(1-q^{r_0+1-s})}{1-q^{-1}} < \frac{\left(e^{\frac{q}{q-1}}\right)^2}{q-1}q^{-r_0} \quad (7)$$

As $q \ge 16$, $\frac{\left(e^{\frac{q}{q-1}}\right)^2}{q-1} < 1$. Thus, $\sum_{r=r_0+1}^{s} \mu_s^t(r) \le q^{-r_0} \le q^{-1}$. Hence, $\mu_s^t(r_0) > 1-q^{-1}$

**Case 2.** $r_0+1 = s$. Then $\mu_s^t(r_0) = 1-\sum_{r=r_0+1}^{s} \mu_s^t(r) = 1-\mu_s^t(r|r = r_0+1)$. By Lemma 8, we can obtain $\mu_s^t(r \mid r = r_0+1) \le q^{-r_0-1}(e^{\frac{q}{q-1}})^2$. As $q \ge 16$, we have $q^{-1}\left(e^{\frac{q}{q-1}}\right)^2 < 1$. Therefore, we obtain $\mu_s^t(r|r = r_0+1) < q^{-r_0} \le q^{-1}$. Hence, $\mu_s^t(r_0) = 1-\mu_s^t(r|r = r_0+1) > 1-q^{-1}$.

**Case 3.** $r_0+1 > s$. As $r_0 \le s$, we obtain $r_0 = s$. Thus, for $t = n$, we obtain $\mu_s^t(r_0) = 1 \ge 1-q^{-1}$.

Altogether, the theorem hods. □

**Corollary 2.** *For any $\alpha \in V_s$, let $l = dim(span(\alpha, V_t))$. (i) If $t = n$, then $Pr(l = t) = 1$. (ii) Otherwise, we have $Pr(l = t+1) > 1-q^{-1}$.*

*Proof*. We still consider a case study.

**Case 1.** $t = n$. Thus, $V_t = V$. As $\alpha \in V_s \subseteq V = V_t$ and $\alpha \in V_t$, $span(\alpha, V_t) = V_t$. Hence $l = t$. That is, $Pr(l = t) = 1$.

**Case 2.** Otherwise, $t < n$. In this case, $r_0 = s+t-n < s$. By Theorem 3, the probability that $V_s$ and $V_t$ have $r_0$ common bases approximately equals to 1. Therefore, $Pr(\alpha \in V_t) = q^{s-r_0} < q^{-1}$. Then we obtain $Pr(l = t+1) = 1-Pr(l = t) = Pr(\alpha \in V_t) > 1-q^{-1}$.

Altogether, the corollary holds. □

# 5. DELAY ANALYSIS

## 5.1. Contact probability

We say two nodes are in contact if they meet each other and win the competition to communicate. In our scenario, packet delivery incurs under three types of contacts:

(1) The source contacts with the destination with probability $I_{\mathrm{SD}}$.
(2) A relay contacts with the source with a probability $I_{\mathrm{RS}}$.
(3) A relay contacts with the destination with a probability $I_{\mathrm{RD}}$.

These three probabilities may be different under the different mobility models. We can derive the expressions for $I_{\mathrm{SD}}$, $I_{\mathrm{RS}}$, and $I_{\mathrm{RD}}$ in our GRW model as follows.

$$I_{\mathrm{RS}} = \frac{2(M^2-1)}{M^4}\sum_{j=0}^{N-1} f\left(j, N-1, \frac{1}{M^2}\right)\frac{1}{(j+1)(j+2)} \quad (8)$$

where $f\left(j, N-1, \frac{1}{M^2}\right) = \binom{N-1}{j}\left(\frac{1}{M^2}\right)^j\left(1-\frac{1}{M^2}\right)^{N-1-j}$. Further, we have

$$I_{\mathrm{RS}} = I_{\mathrm{RD}} \quad (9)$$

$$I_{\mathrm{SD}} = \frac{1}{M^2} \quad (10)$$

Let us explain more of $I_{\mathrm{RS}}$. When one selected relay contacts with the source, it means that the following three independent events occur simultaneously. (i) This selected relay and any other $j (0 \le j \le N-1)$ relays are in the same cell the source located, and (ii)the destination is not in this cell. Thus, there are $j+2$ nodes (the source, the selected relay, and $j$ other relays) nodes in this cell. (iii) The source and this relay win the contact chance, whereas these $j+2$ nodes compete for the communication chance. Recall that the stationary distribution of our GRW mobility model is uniform a uniform distribution. The probability of these three events is $\frac{1}{M^2}\sum_{j=0}^{N-1} f\left(j, N-1, \frac{1}{M^2}\right), 1-\frac{1}{M^2}$, and $\frac{1}{\binom{j+2}{2}}$. So we can obtain the expression (8) easily.

## 5.2. Update of innovativeness rank

Recall that the innovativeness rank of any node indicates how many innovative packets this node holds w.r.t. the destination. Thus, for any relay $R_i$, its innovativeness rank $IR_i$ might change in three cases. (i) If $R_i$ contacts with the source, $IR_i$ might increase by 1. (ii) If $R_i$ contacts with the destination, $IR_i$ might decrease by 1. (iii) If other relay, say $R_j$, deliveries a packet to the destination, which might change the destination's rank, $IR_i$ also might decrease by 1.

To be more clear, we suppose that at time $T$ the destination $R_{N+2}$ has received $d(T)$ innovative packets, namely $rank(R_{N+2}) = d(T)$. Let $n(T) = K-d(T) \ge 1$. Obviously at this time, $R_{N+2}$ requires $n(T)$ packets to receive. We denote the spaces spanned by these $n(T)$ packets as

$RV_n(T)$. For any two relays $R_i$ and $R_j$ whose innovativeness rank are $IR_i(T)$ and $IR_j(T)$, respectively, we denote the spaces spanned by innovative packets in their buffer as $RV_i(T)$ and $RV_j(T)$, respectively. It is obvious that $RV_i(T)$ and $RV_j(T)$ are two subspaces of $RV_n(T)$. Using the results in Theorem 3 and Corollary 2, we can obtain the following conclusions:

**Conclusion 1.** *Suppose at time $T+1$ the source $R_{N+1}$ sends a packet to $R_i$. If $IR_i(T) < n(T)$, then $Pr(IR_i(T+1) = IR_i(T)+1) = 1 - q^{-n(T)}$ (This value is approximately equal to 1 when $q$ is sufficiently large). Otherwise, $IR_i(T+1) = IR_i(T)$.*

**Conclusion 2.** *Suppose at time $T+1$ the relay $R_i$ sends a packet to the destination $R_{N+2}$ (which hints $IR_i(T) \geq 1$), then $Pr(d(T+1) = d(T)+1) = 1-q^{-IR_i(T)}$ (This value is approximately equal to 1 when $q$ is sufficiently large) and $IR_i(T+1) = IR_i(T) - 1$.*

**Conclusion 3.** *When the relay $R_i$ sends a packet to the destination $R_{N+2}$, $IR_j(T+1)$ also might change. Let $\delta(T) = \min\{IR_i(T), n(T) - IR_j(T)\}$. If $IR_j(T) < n(T)$, then $Pr(IR_j(T+1) = IR_j(T)) = 1-q^{-\delta(T)}$ (This value is approximately equal to 1 when $q$ is sufficiently large.) Otherwise, $IR_j(T+1) = IR_j(T) - 1$.*

To conclude, suppose the size of Galois field $q$ is sufficiently large (which is 256 as in our simulation); when one relay $R_i$ sends (or receives) a packet, its innovativeness rank may change. However, the innovativeness rank of other relay $R_j$'s does not change unless $R_j$ has all the innovative packets that the destination needs. As shown in Figure 2(a), at time $T$, the destination $R_{N+2}$ has four independent packets, and the innovativeness ranks of relay $R_i$, relay $R_j$, and relay $R_m$ are 3, 4, and 2, respectively. Suppose $R_i$ delivers a packet to the destination successfully. After that, the destination has five independent packets, and the innovativeness ranks of $R_i$, $R_j$, and $R_m$ are 2, 3, and 2, respectively, as shown in Figure 2(b). That is, sending a packet to the destination by $R_i$ results in the decrease of $R_j$'s innovativeness rank, but no change on that of $R_m$'s.

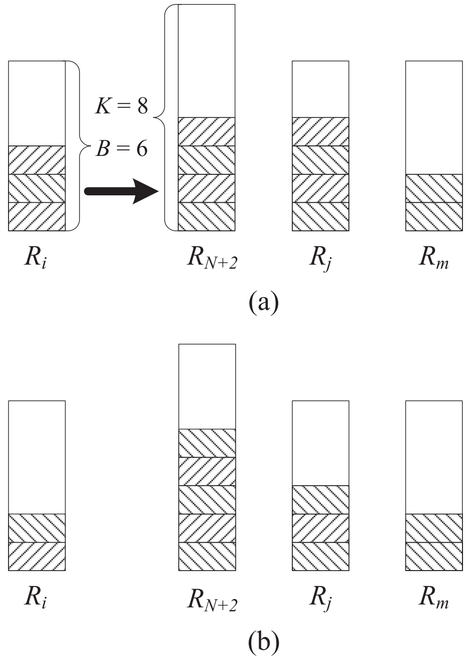


**Figure 2.** The change of node's innovativeness rank.

## 5.3. Network state

We define the network state as $\{X_0(T), X_1(T), \ldots X_i(T) \ldots X_B(T)\}$, where $X_i(T)$ denotes the number of relay nodes whose innovativeness rank is $i$ at time $T$. Thus, we classify relays into $B+1$ categories. Each node belonging to $X_i$ category is called type $i$ node. It is obvious that $\{X_0(T), X_1(T), \ldots, X_i(T), \ldots X_B(T)\}$ is a Markov chain. This network state definition resembles the one in [25]. However, in [25], authors care about the rank of relays' coefficient matrix, whereas we care about the relative innovativeness rank of a node.

Now, we can explore the nonzero transition rates of this chain. By the update rules of node's innovativeness rank, normally, a type $i$ node will become a type $i-1$ node when it contacts the destination. Thus, its innovativeness rank decreases by 1. Further, it becomes a type $i+1$ node when it receives a packet from the source, and hence its innovativeness rank increases by 1. For an empty node, it cannot send packets to the destination, whereas for a type $B$ node, it cannot receive a packet from the source. Thus, we have

$$
\begin{aligned}
&X_0 \to X_0 - 1 \text{ at rate } I_{\mathrm{RS}} X_0,\\
&X_0 \to X_0 + 1 \text{ at rate } I_{\mathrm{RD}} X_1,\\
&X_i \to X_i - 1 \text{ at rate } (I_{\mathrm{RS}} + I_{\mathrm{RD}}) X_i,\\
&X_i \to X_i + 1 \text{ at rate } I_{\mathrm{RS}} X_{i-1} + I_{\mathrm{RD}} X_{i+1},\\
&X_B \to X_B - 1 \text{ at rate } I_{\mathrm{RD}} X_B,\\
&X_B \to X_B + 1 \text{ at rate } I_{\mathrm{RS}} X_{B-1}.
\end{aligned}
$$

Suppose that at time $T$ the destination has already received $d(T)$ independent packets from the network. Then for any relay node, its innovativeness rank cannot be larger than $\min\{B, K-d(T)\}$. Thus, if a relay node has more than $\min\{B, K-d(T)\}$ innovative packets, it will become a type $\min\{B, K-d(T)\}-1$ node. Thus, we can calculate $X_i(T)$ iteratively as follows. The equation for $d(T)$ is shown in expression (19).

Let $IM(T) = \min(B, \lfloor K - d(T) \rfloor)$, and $\Delta X_{i-1}^{i}(T) = X_i(T) - X_{i-1}(T)$, where $i = 1, \ldots, B$. Then we have

$$X_0(T+1) = X_0(T) - I_{\mathrm{RS}} X_0(T) + I_{\mathrm{RD}} X_1(T) \tag{11}$$

$$
\begin{aligned}
X_i(T+1) = X_i(T) &+ I_{\mathrm{RD}} \Delta X_i^{i+1}(T)\\
&- I_{\mathrm{RS}} \Delta X_{i-1}^{i}(T), \quad \text{for } 1 \leq i < IM(T)
\end{aligned} \tag{12}
$$

$$X_i(T+1) = 0, \quad \text{for } IM(T) < i \le B \tag{13}$$

$$X_{IM(T)}(T) = X_{IM(T)}(T) + X_{IM(T)+1}(T) + I_{\mathrm{RD}} \Delta X_{IM(T)}^{IM(T)+1}(T) - I_{\mathrm{RS}} \Delta X_{IM(T)-1}^{IM(T)}(T), \quad \text{if } IM(T) < B \tag{14}$$

$$X_{IM(T)}(T) = X_B(T) - I_{\mathrm{RD}} X_B(T)] + I_{\mathrm{RS}} X_{B-1}(T), \quad \text{if } IM(T) = B \tag{15}$$

The initial value of $\{X_0, \ldots, X_i, \ldots, X_B\}$ is given by $\{X_0(0), \ldots, X_i(0), \ldots, X_B(0)\} = \{N, \ldots, 0, \ldots, 0\}$ because at the start of the block transfer (i.e., $T = 0$) none of the relay nodes have any innovative packet.

### 5.4. Delay distribution approximation

With the preceding theoretical results, we can draw the conclusion that, approximately, the destination, when it has less than $K$ packets, can always obtain an innovative packet each time when it contacts with any nonempty relay or the source. That is, we can compute the rate that the destination receives packets when it already has $i$ packets

$$D_i = (N - X_0(T)) I_{\mathrm{RD}} + I_{\mathrm{SD}} \quad \text{for} \quad i = 0, 1, \ldots K-1 \tag{16}$$

Thus, let $T_i$ be the delivery delay, that is, the time from the beginning to the time when the destination receives $i$ innovative packets from the network successfully. We denote the cumulative distribution function (CDF) of $T_i$ by $P_i^N(T) = Pr(T_i < T)$ while there are $N$ relay nodes. As shown in Appendix B, we have

$$P_1^N(T+1) = \left(1 - P_1^N(T)\right) [(N - X_0(T)) I_{\mathrm{RD}} + I_{\mathrm{SD}}] \tag{17}$$

$$P_i^N(T+1) = P_i^N(T) + [(N - X_0(T)) I_{\mathrm{RD}} + I_{\mathrm{SD}}] \left(P_{i-1}^N(T) - P_i^N(T)\right), \quad \text{for } i = 2, \ldots, K \tag{18}$$

The initial value for every $P_i^N(T)$ is given by $P_i^N(0) = 0$, where $i = 1, \ldots, K$.

We derive $d(T)$ by

$$d(T) = \max\left(i \,|\, P_i^N(T) = 1\right) \tag{19}$$

## 6. EXPERIMENTAL VALIDATION

In this section, we first validate our analysis on the packet dependency due to the overlap between packets' spaces. Then we present the comparison of the block delivery delay between our analysis and the simulations.

### 6.1. The packets dependency

We want to validate the probabilities of the following four events by simulations and compare them to our analysis in Sections 4 and 5.1.

**Event 1.** *Although $d(T) < K$, each packet that the source $R_{N+1}$ sends out is innovative to the destination. With Conclusion 1 of our analysis in Section 5.2, the probability of this event should be approximately equal to 1.*

**Event 2.** *Although $d(T) < K$, each packet that any relay $R_i$ (whose innovativeness rank is positive) sends out to the destination $R_{N+2}$ is innovative to the destination. With Conclusion 2 of our analysis in Section 5.2, the probability of this event should be approximately equal to 1.*

**Event 3.** *Consider any two relays $R_i$ and $R_j$ both of which have innovativeness rank $IR_i$ and $IR_j$, respectively,*

**Table I.** Probability of Events 1 and 2.

| $d(T)$ | $Pr$ (Event 1) | $Pr$ (Event 2) |
|---|---|---|
| 0 | 0.9948 | 0.9975 |
| 1 | 0.9895 | 0.9968 |
| 2 | 0.9876 | 0.9979 |
| 3 | 0.9864 | 0.9972 |
| 4 | 0.9866 | 0.9956 |
| 5 | 0.9998 | 0.9954 |
| 6 | 1.0000 | 0.9977 |
| 7 | 0.9996 | 0.9966 |
| 8 | 0.9989 | 0.9987 |
| 9 | 0.9961 | 0.9934 |

**Table II.** Probability of Event 3.

| | $IR_j$ | | | | |
|---|---|---|---|---|---|
| $IR_i$ | 1 | 2 | 3 | 4 | 5 |
| 0 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 1 | 0.999 | 0.998 | 1.000 | 1.000 | 1.000 |
| 2 | 0.998 | 1.000 | 0.998 | 1.000 | 1.000 |
| 3 | 0.996 | 1.000 | 1.000 | 1.000 | 1.000 |
| 4 | 0.997 | 0.994 | 1.000 | 1.000 | 1.000 |
| 5 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

**Table III.** Probability of Event 4.

| | $IR_j$ | | | | |
|---|---|---|---|---|---|
| $IR_i$ | 1 | 2 | 3 | 4 | 5 |
| 0 | — | — | — | — | — |
| 1 | 0.988 | — | — | — | — |
| 2 | 1.000 | 1.000 | — | — | — |
| 3 | 1.000 | 1.000 | 1.000 | — | — |
| 4 | 1.000 | 1.000 | 1.000 | 1.000 | — |
| 5 | 0.995 | 1.000 | 1.000 | 1.000 | 1.000 |


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DOI: 10.1002/wcm

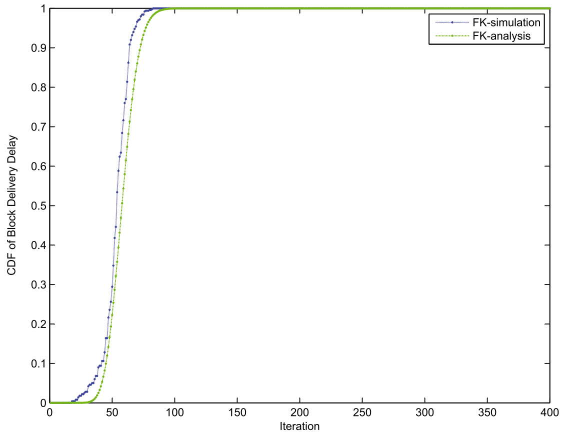


**Figure 3.** CDF of block delivery delay ($K = 10$, $N = 50$, $B = 5$, $M = 5$).

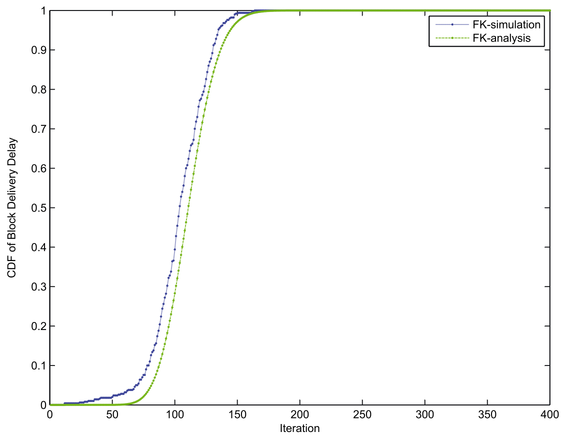


**Figure 4.** CDF of block delivery delay ($K = 10$, $N = 200$, $B = 5$, $M = 10$).

*which satisfy* $0 \leq IR_j(T) < K - d(T)$ *and* $R_i(T) > 0$*. Although* $d(T) < K$*, when* $R_i$ *sends a packet to the destination* $R_{N+2}$*,* $R_j$*'s innovativeness rank remains unchange. With Conclusion 3 of our analysis in Section 5.2, the probability of this event should be approximately equal to 1.*

**Event 4.** *Consider any two relays* $R_i$ *and* $R_j$ *both of which have innovativeness rank* $IR_i$ *and* $IR_j$*, which satisfy* $IR_i(T) > 0$ *and* $IR_j(T) = K - d(T)$*. Although* $d(T) < K$*, when* $R_i$ *sends a packet to the destination* $R_{N+2}$*,* $R_j$*'s innovativeness rank decreases by 1. With Conclusion 3 of our analysis in Section 5.2, the probability of this event should be approximately equal to 1.*

In this part, we set the number of relay nodes as 200 ($N = 200$), the number of packets to be delivered as 10 ($K = 10$), the grid size as 10 ($M = 10$), and the buffer size of relays as 5 ($B = 5$). To achieve more reasonable results, 5000 simulations are conducted. Table I shows the probabilities of events 1 and 2. For events 3 and 4, we obtain the probability according to different innovativeness ranks $IR_i$ of $R_i$ and $IR_j$ of $R_j$, as shown in Tables II and III.

As can be seen from the simulation results in Tables I–III, the probabilities of the preceding four events are approximately equal to 1, which coincide with our analysis in previous sections. We examined some other scenarios and obtained similar results, but we omit them for brevity.

### 6.2. Block delivery delay

To verify how accurate our analysis on the block delivery delay is, we set up two network scenarios. In both scenarios, we set the number of packets to be delivered as 10 ($K = 10$). In one scenario, the number of relay nodes is 200 ($N = 50$), the grid size is 5 ($M = 5$), and the buffer size of relays is 1 ($B = 1$). In another scenario, the number of relay nodes is 200 ($N = 200$), the grid size is 10 ($M = 10$), and the buffer size of relays is 5 ($B = 5$). We did 500 simulations under each scenario and took the average. As can be seen from Figures 3 and 4, the two plots obtained by simulation and our analysis are very close to each other for both scenarios. We also examined some other scenarios and concluded the same, which are omitted to avoid redundancy.

## 7. CONCLUSION

In this paper, we analyzed the block delivery delay in DTN for RLNC-based two-hop routing in unicast. Because of the network coding in packet delivery, the dependency among packets in different nodes arises. These dependencies make it more difficult to analyze the packet delivery delay accurately. First, we analyzed packet dependency by a stochastic method. Specifically, packets carried by a node span a space. We accurately calculated the probability that the intersection of two spaces spanned by any pair of relay nodes has different dimensions. Then we simplify this probability by reasonable approximation while the size of Galois field $q$ is sufficiently large, for example, $q \geq 16$, which is very common in reality. Thus, we could track each node's innovativeness rank accurately. Second, we developed an analytic framework to iteratively estimate the CDF of the block delivery delay by tracking of nodes' innovativeness ranks. The simulation results showed that our calculation of dependencies does result in good estimates of nodes' innovativeness ranks. Further, we showed that the CDF derivations of block deliver delay is consistent to the simulation results.

For RLNC-based routing schemes that involve multi-hop unicast (i.e., allowing relay to relay packet exchange), the analysis of dependency is more complicated and will highly influence the performance analysis. Our future work will extend the analysis to more general RLNC-based routing schemes.

## APPENDIX A: THE STATIONARY DISTRIBUTION OF THE GRW MODEL IS UNIFORM

It is almost obvious that the stationary distribution of GRW mobility model is a uniform distribution, as the probability that a node moves out of a grid cell is exactly equal to the summation of the probabilities that it moves in from a neighbor cell.

Specifically, let $n_{ij} = (i-1)M + j$. The mobility of each node in GRW mobility model is characterized by a Markov chain. The states of the Markov chain are $s_{ij}$, meaning that the node is located at the $i$th row and the $j$th column of the grid, $1 \leq i, j \leq M$. The transition probability $P_{n_{ij},n_{kl}}$ from state $s_{ij}$ to state $s_{kl}$ is as follows:

$$P_{n_{ij},n_{kl}} = \begin{cases} \frac{1}{5} & \text{if } (|i-k| = 1 \text{ and } j = l), \\ & (|j-l| = 1 \text{ and } i = k), \\ & \text{or } (1 < i = k < M \text{ and } j = l < M); \\ \frac{2}{5} & \text{if } (i = k = 1 \text{ and } 1 < j = l < M), \\ & (i = k = M \text{ and } 1 < j = l < M), \\ & (1 < i = k < M \text{ and } j = l = 1), \\ & \text{or } (1 < i = k < M \text{ and } j = l = M); \\ \frac{3}{5} & \text{if } (i = j = k = l = 1), \\ & (i = k = 1 \text{ and } j = l = M), \\ & (i = k = M \text{ and } j = l = 1), \\ & \text{or } (i = j = k = l = M); \\ 0 & \text{otherwise.} \end{cases} \tag{A1}$$

One can check straightforward that for any $1 \leq k, l \leq M$, $\sum_{1 \leq i,j \leq M} P_{n_{ij},n_{kl}} = 1$. This means that the $n$-dimensional all-one vector $\pi = (1, 1, \ldots 1)$ satisfies $\pi = \pi P$. As a result, uniform distribution is a stationary distribution of the GRW model.

## APPENDIX B: DERIVATION OF FORMULAS (17) AND (18)

It is easy to see that the event $\{T_i \geq T + 1\}$ implies the event $\{T_i \geq T\}$. Hence, we have $\{T_i \geq T + 1, T_i \geq T\} = \{T_i \geq T + 1\}$ and

$$\begin{aligned} Pr(T_i \geq T + 1) &= Pr(T_i \geq T + 1, T_i \geq T) \\ &= Pr(Ti \geq T) Pr(T_i \geq T + 1 | T_i \geq T) \\ &= Pr(T_i \geq T)(1 - Pr(T_i < T + 1 | T_i \geq T) \end{aligned} \tag{B1}$$

Next, we derive $Pr(T_i < T + 1 | T_i \geq T)$. In a time slot, the destination can obtain at most one packet. Hence, the event $\{T_i < T + 1\}$ that the destination receives $i$ packets before time slot $T+1$ happens only if the destination receives $i-1$ packets before time slot $T$, that is, the event $\{T_{i-1} < T\}$. Therefore, we have

$$Pr(T_i < T + 1 | T_i \geq T) \simeq D_{i-1} Pr(T_{i-1} < T | T_i \geq T) \tag{B2}$$

where $D_{i-1}$ is the receiving rate of the destination when it has $i-1$ packets, which can be computed via formula (16). We then derive $Pr(T_{i-1} < T | T_i \geq T)$ as follows:

$$\begin{aligned} Pr(T_{i-1} < T | T_i \geq T) &= 1 - Pr(T_{i-1} \geq T | T_i \geq T) \\ &= 1 - \frac{Pr(T_{i-1} \geq T, T_i \geq T)}{Pr(T_i \geq T)} \\ &= 1 - \frac{Pr(T_{i-1} \geq T)}{Pr(T_i \geq T)} \end{aligned} \tag{B3}$$

where the third equality holds as $T_i \geq T_{i-1}$, the time to receive $i$ coded packets is always greater than or equal to the time to receive $i-1$ coded packets. Substituting (B3) into (B2), and (B2) into (B1), we obtain

$$\begin{aligned} Pr(T_i \geq T + 1) = Pr(T_i \geq T) + \\ D_{i-1}(T)(Pr(T_{i-1} \geq T) - Pr(T_i \geq T)) \end{aligned} \tag{B4}$$

Therefore, we can compute the derivative of $P_i^N(T + 1)$ by

$$\begin{aligned} P_i^N(T + 1) - P_i^N(T) &= Pr(T_i < T + 1) - Pr(T_i < T) \\ &= Pr(T_i \geq T) - Pr(T_i \geq T + 1) \\ &= (Pr(T_i \geq T) \\ &\quad - Pr(T_{i-1} \geq T)) D_{i-1} \end{aligned} \tag{B5}$$

Thus,

$$\begin{aligned} P_i^N(T + 1) = P_i^N(T) + (Pr(T_i \geq T) \\ - Pr(T_{i-1} \geq T)) D_{i-1} \end{aligned} \tag{B6}$$

By substituting (16) into (B5), we obtain formula (18). Similarly, we can derive formula (17).

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