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Network coding for multiple unicast sessions in multi-channel/interface wireless networks

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Abstract Throughput limitation of wireless networks imposes many practical problems as a result of wireless media broadcast nature. The solutions of the problem are mainly categorized in two groups; the use of multiple orthogonal channels and network coding (NC). The networks with multiple orthogonal channels and possibly multiple interfaces can mitigate co-channel interference among nodes. However, efficient assignment of channels to the available network interfaces is a major problem for network designers. Existing heuristic and theoretical work unanimously focused on joint design of channel assignment with the conventional transport/IP/MAC architecture. Furthermore, NC has been a prominent approach to improve the throughput of unicast traffic in wireless multi-hop networks through opportunistic NC. In this paper we seek a collaboration scheme for NC in multi-channel/interface wireless networks, i.e., the integration of NC, routing and channel assignment problem. First, we extend the NC for multiple unicast sessions to involve both COPE-type and a new proposed scheme named as Star-NC. Then, we propose an analytical framework that jointly optimizes the problem of routing, channel assignment and NC. Our theoretical formulation via a linear programming provides a method for finding source–destination routes and utilizing the best choices of different NC schemes to maximize the aggregate throughput. Through this LP, we propose a novel channel assignment algorithm that is aware of both coding opportunities and co-channel interference. Finally, we evaluate our model for various networks, traffic models, routing and coding strategies over coding-oblivious routing.

Keywords Network coding · Multi-channel/interface · Channel assignment · Unicast routing · Wireless Mesh Network

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1 Introduction

Wireless networks provide means for mobility, internet connectivity and distributed sensing. However, the throughput limitation of these networks imposes many practical problems. Two different mechanisms to increase the utilization of wireless network are network coding (NC) and coexistence of multiple orthogonal channels. Both trends have triggered a large body of work on designing wireless networks with a larger capacity. The former leads to design multi-channel wireless networks (especially multi-hop wireless mesh networks-WMN) with multiple interfaces to reduce co-channel interference while the second tries to redesign the network layer of protocol stack to achieve the capacity identified by max-flow min-cut theorem.

Mounting multiple interfaces on one 802.11 device leads to a lot of researches on designing multi-channel wireless mesh networks with multiple interfaces. Since the number of orthogonal channels are limited (3 in 802.11b/g and 12 in 802.11a), the key problem is to assign appropriate channels to the interfaces on each node in a way that the interference between neighboring links with overlapping channels is reduced as long as the network remains connected, thereby maximizing the network capacity. Most existing work focused on incorporating the channel/interface design problem into the traditional network architecture, proposing joint design with routing [1–3], with topology control [4], with MAC protocol [5, 6], as well as with congestion control [7]. The perspectives of existing problem formulations include both optimization based theoretical studies and heuristic based protocol design. However, most of them are hold in conventional network protocol stacks.

On the other hand, NC, proposed in [8], is an information theoretic approach to increase throughput of wireless networks by employing the broadcast nature of wireless medium. In the work of Li et al. [9], it was shown that linear codes are sufficient to achieve maximum throughput for multicast traffic. Unicast, on the other hand, has received less attention relative to the multicast. In [10–13], specific topologies with unicast routing were studied and shown that NC results better throughput than standard routing. In [14], COPE was introduced, which is a packet encoding scheme via XOR operation. The authors studied certain basic topologies such as chain, cross and wheel in a unicast traffic model and reported the throughput gain as the first testbed deployment of wireless NC. In the follow up, Sengupta et al. [15] extended the use of COPE in wireless network with any patterns of multiple concurrent unicast transmissions. From a theoretical perspective, the authors provided two linear programming formulations for measuring the throughput improvements of COPE-type NC scheme for both with and without opportunistic listening. By these formulations, the authors advocated the idea of *coding-aware routing*, i.e., the routing which selects the paths with the awareness of NC opportunities.

Both trends share a common objective, incorporating the routing into the original channel assignment/network coding problem. More specifically, as mentioned above, for NC in unicast transmissions, *coding-aware routing* refers to the selection of routes that provides more NC opportunities [15]. Moreover, most work on joint NC and routing optimization considers the idea of multi-path routing [15, 16]. It takes *interference-aware* routing into account, i.e., the paths that minimize the interference are selected. In addition to multi-path, a scenario so called optimized single-path was proposed in [16]. It presents an *interference-aware* single-path routing. In a recent work [17], Zhang proposed "optimized multipath network coding" (OMNC), a rate control and routing protocol that improves the throughput of lossy wireless networks.

On the other hand, routing and channel assignment are strictly inter-dependent. This is due to the impact of channel assignment on link bandwidths and the extent to which link retransmissions interfere. Similarly, routing of a desired traffic determines traffic flows for each link which certainly affects channel assignment. Since the problem of joint routing and channel assignment is generally NP hard, the solution is either a heuristic algorithm or an optimization based approximation algorithm. The proposed algorithms in [3] and [6] are instances of heuristic approaches on channel assignment and load-aware routing to improve the aggregate throughput of WMNs. The main idea is to define a new routing metric using the specific properties of WMNs. In contrast, the approach in [1] and [18] is based on a linear programming formulation, and used a centralized approximation algorithm to derive the feasible solution. This formulation is the base of the future work such as [16] and [19].

In this paper we consider the joint channel assignment and routing optimization problem with the addition of NC with various coding structures. More specifically, we study the following set of issues:

1.1 The benefits of star-structure and COPE-type network coding

In this paper, we propose a new NC scheme based on the opportunistic listening with a new concept of *star-structure network coding*. Our scheme, referred as Star-NC, considers multiple unicast transmission flows intersecting each other at a relay node. The relay node can decrease its transmissions by mixing the packets belong to the corresponding flows (sessions). This reduction is due to the opportunistic listening of the nodes at the proximity of the relay node. The key idea of our scheme is generality and flexibility of the opportunistic listening which is done among the nodes around the relay node. Our extended form of overhearing creates more coding opportunities which in consequence lets more reduction in transmissions of the relay nodes of the network.

We focus on the star structures as a primitive element in the unicast traffic and propose an NC scheme for such structures. In an arbitrary network, each node with its neighbors creates a star structure. Thus, we can consider the star structures as primary elements of wireless networks. A node with a larger degree has more chance to be the cross point of some unicast sessions and acts as a relay node wherein an opportunity of Star-NC is created. The results show that Star-NC provides new coding opportunities different from COPE-type NC and thus can be used either with or without COPE. Moreover, it states that Star-NC has often better performance than COPE for *directional* traffic models.

In this paper, we extend the concept of Star-NC to multi-channel/interface network. Along with, we consider COPE-type NC scheme for both with and without opportunistic listening paradigms. We further take the joint of Star and COPE type NC schemes into account. We provide a theoretical framework which evaluates the throughput improvement of a specific NC scheme for a given topology, a channel assignment, a set of traffic demands, and a routing strategy (e.g., with shortest-hop or WCETT[1] [20] metric) relative to non-coding scheme.

Effect of routing strategies Clearly, NC in multi-channel system has a trade-off between *interference-aware* and *coding-aware* routing. The former is the result of the routing choices that mitigate interference among the neighbor nodes while the latter arises from the selection the paths that facilitates more coding opportunities. Thus, in this paper, we try to present a systematic approach for choosing routes that optimize the tradeoffs between the conflicting effects of increased coding opportunities and increased wireless interference. Moreover, as mentioned before, both multi-path and optimized single-path routing take *interference-aware* routing into account and accordingly are able to better utilize the capacity of a given network. Thus we study the benefits of different NC schemes for both multi-path and optimized single-path in addition to single-path routing. Moreover, as the network is multi-channel/interface, a routing metric must be selected that is able to boost the capacity of emerging multi-channel systems. A suitable metric is the one which takes the channel switching among a selected path into account. Therefore, we choose WCETT as the routing metric since it considers both the delay and channel diversity of the path. Furthermore, WCETT acts like as shortest path routing, under proper parameterization.

Effect of traffic model We consider a *directional traffic* model in addition to *random traffic* model. Directional traffic refers to a traffic model in which the source nodes are apart from the destination nodes, i.e., the set of senders and receivers are disjoint. We use this model as it suits in WMNs that provide Internet connectivity to end users. The results show that traffic model has impact on coding opportunities created either by COPE-type NC or Star-NC scheme, thereby is a base for comparing capabilities of these coding schemes.

Coding-aware and interference-aware channel assignment There is a fundamental trade-off between the channel diversity of the links that are within interference range of each other and the channel uniformity of them that create a coding opportunity. To handle this trade-off, we formulate an optimization problem that jointly optimizes routing, channel assignment and NC. Since this problem is NP-hard in general, we design a simulated annealing [21] based algorithm to touch the optimal solution. The proposed channel assignment is a closed-loop algorithm with a random initialization that iteratively updates channel frequencies of the nodes using the link load information evaluated by the LP formulation. The main idea of iteration is to find a single link, within the interference range of all of heavy load links, in order that switching its channel frequency to a new one leads to a better aggregate throughput. Such single-link tuning conducts a gradual movement from an initial channel assignment state to near optimal state.

1.2 Prior work on NC in multi-channel system

As prior and related work, we can point to [16, 19, 22] and [23]. The authors in [19] and [16], studied the benefits of NC in multi-channel wireless networks. In both of them, the coding scheme is limited to the simple COPE-type NC without opportunistic listening. Further, an LP formulation is used to maximize the throughput for a specific traffic demands in a given network topology. Further, based on the LP formulation, a randomized channel assignment algorithm was proposed in [19]. The formulation in [16] employs a practically infeasible coding at physical layer, named as *analog NC*, that mixes simultaneously arrived radio waves in analog domain. The authors also developed a heuristic joint link scheduling, channel assignment, and routing algorithm that aims at approaching the optimal solution of the optimization problem. Furthermore, as a heuristic approach in the absence of LP formulation, [22] and [23] studied the benefits of NC with opportunistic listening in a multi channel/interface network. The proposed scheme in [23] is based on the combination of the *coded-overhearing* and *coding-aware* channel assignment. The proposed algorithm overcomes the radio coverage limitations in conventional NC schemes, and improves the aggregate throughput when the nodes do not have sufficient number of interfaces.

As a comparison of our work with the above work, we can point out several differences including the NC schemes, routing methods and channel assignment approaches. Instead of employing only a simple packet exchange NC scheme in [19] and [16], we study various coding structures either with or without opportunistic listening. Further, compare to [22] and [23] which have a heuristic approach with a restricted modeling of network topology, traffic, routing strategies and coding scheme, we have an optimization based approximation algorithm with a more extensive modeling. In summary, our formulation covers general forms for network topology, traffic models, routing strategies and coding schemes.

1.3 Contributions of the work

The main contributions of this paper can be summarized as follows:

[1] Weighted cumulative expected transmission time.

- We study various NC schemes in multi-channel system. First, we consider COPE-type NC for both opportunistic listening and hybrid paradigms in addition to information exchange paradigm. Second, we study Star-NC, a new NC for multiple unicast sessions based on the opportunistic listening. The proposed NC scheme has an extended form of opportunistic listening relative to COPE-Type NC scheme. This feature allows our scheme for creating more coding opportunities based on the overhearing information. Specifically, the benefit of Star-NC appears in directional traffic model which is a typical form of traffic for wireless mesh networks that provides Internet connectivity.
- We provide a linear programming formulation to evaluate throughput performance of Star-NC scheme for any configuration of wireless network, channel assignment, traffic model and routing method. Our formulation allows the integration of our NC scheme with other coding schemes such as COPE.
- We propose a coding-aware and interference-aware channel assignment algorithm, i.e., assigns channels to the interfaces of each node in order to facilitate more coding opportunities in addition to mitigate interference in the network. Through this, we provide a collaboration scheme of NC, routing and channel assignment in multi-hop WMNs.

The rest of this article is organized as follows. In Sect. 2, we present a background for NC for multiple unicast sessions. In particular, COPE-type coding scheme along with a new coding scheme, named as Star-NC, is introduced. Next, in Sect. 2, we formulate the problem of NC in multi-channel system for the mentioned NC schemes. In Sect. 4, we develop a theoretical formulation to study the benefits of different NC schemes over non-coding schemes. In Sect. 5, using the formulation, we propose our coding-aware and interference-aware channel assignment algorithm. Then, in Sect. 6, we evaluate the benefit of the collaboration scheme with various network topologies, traffic models and routing strategies. Finally, Sect. 7 concludes the paper.

2 Background and related work

We are concerned with the unicast transmissions between any two nodes in the desired wireless network. When two or more unicast transmissions cross each other, a coding opportunity at the cross point (referred as the relay node) is created. Based on the number of unicast transmissions and the way of crossing, various structures and topologies can be generated. We study these structures in two basic categories. The first is the COPE-type structure which is a two-hop coding scheme [14, 15]. The second is star-structure NC which is a new two/three-hop coding scheme. In this section, first we introduce the concept of COPE-type NC. Next, a concept of star structure NC, namely Star-NC, is introduced.

2.1 COPE-type network coding

COPE which uses XOR operation to perform coding, has two interesting features:

1) Opportunistic coding: Each wireless node uses only the packets in its local queues for coding. This allows taking the advantage of NC through local decisions without requiring any form of global coordination among different nodes.
2) Opportunistic listening: Exploiting the broadcast nature of the wireless medium, COPE allows the nodes to overhear all of the packets communicated by its neighbors. The overheard packets are used in the coding decisions.

In general, NC opportunities have three types of scenarios: the *information exchange*, the *opportunistic listening* and the *hybrid* paradigm. These paradigms for COPE-type NC are respectively shown in Fig. 1(a), (b) and (c). The information exchange refers to a situation which some nodes around the relay node have packets to destination of each others. The scheme in Fig. 1(a) shows this situation for two nodes around a relay node which is known as Alice-Bob topology. Lemma 1 of [15] states that this is the only scenario for packet exchange paradigm, i.e., the scenario is only occurred for two nodes around a relay node. The opportunistic listening paradigm with four nodes is shown in Fig. 1(b) wherein D_1/D_2 can overhear transmission of S_2/S_1. This scheme is known as X topology. The hybrid paradigm, depicted in Fig. 1(c) with three nodes, takes the advantage of both packet exchange and opportunistic listening. Sengupta et al. [15] introduced the concept of *coding structure* to formulize different scenarios for COPE-type NC scheme.

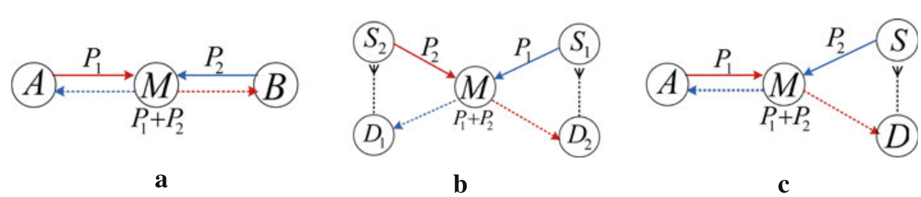

Fig. 1 The COPE-type NC schemes. **a** Packet Exchange paradigm (Alice-Bob Topology), **b**. Opportunistic listening paradigm (“X” topology), **c**. Hybrid paradigm

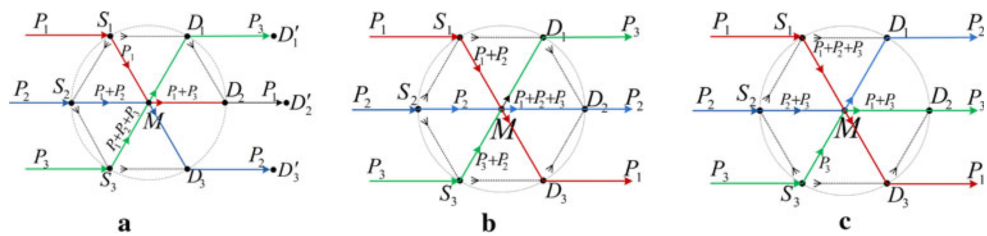

Fig. 2 The full Star-NC schemes of size 3. **a**. $\pi = (3,1,2)$, **b**. $\pi = (3,2,1)$, **c**. $\pi = (2,3,1)$

2.2 Star-structure network coding

Star-NC is centered on star structure as a basic element of NC in wireless networks. A star structure of size n is composed of n input nodes, a relay node and n output nodes in which the packets of input nodes must be forwarded to output nodes by the relay node. In other words, the relay node is the cross point of n different paths corresponding to n unicast transmissions in the network. This structure acts as a switch in the network and forwards the receiving packets to next hop nodes as an intermediate step of routing. The basic idea was explained in Figs. 2 and 3 wherein the transmitted and overheard packets are indicated by the solid and dotted lines, respectively. We denote the input nodes by S_1 through S_n, the output nodes by D_1 through D_n and the relay node by M. The schemes in Fig. 2 are *full-star* structures while Fig. 3 depicts *partial-star* structures.

We identify the full-star from partial-star with a concept of *overhearing direction*. It refers to the direction of overheard node to listener node relative to the relay node. In full-star structure, such as Fig. 2(a), we can see overhearing of both clockwise and counter clockwise directions. However, in partial-star structure the overhearing is limited to be in one direction, e.g. clockwise in Fig. 3(a). This limitation can be imposed by the geographical position of the nodes around the relay node. In full-star, the first output node is within the transmission range of the first input node. The same is true for the last output and input nodes. However, in the partial-star, only the first input and output nodes are within the transmission range of each other. Indeed, the last output node is far from any of the input nodes and, hence, does not have a direct link to any of them.

The goal is to route the incoming packets of input nodes to corresponding output nodes. For instance, in Fig. 2 (a), the incoming packets (P_1, P_2, P_3) must be routed, in sequence, to (D_2, D_3, D_1). The same must be occurred for the incoming packets (P_1, P_2) in Fig. 3(a) and the nodes (D_2, D_1). Indeed, these packets belong to different independent unicast sessions which include the corresponding paths [e.g., S_1–M–D_2, S_2–M–D_3, S_3–M–D_1 in Fig. 2(a)]. In standard routing, this is done by sending n packets via the relay node M to the output nodes. However, as depicted, star structure NC can do this by sending a single coded packet to the output nodes through node M for the schemes depicted in Fig. 2. Therefore, two transmissions are saved for M as a relay node in these schemes. Similarly, one transmission is saved by each of the NC schemes in Fig. 3.

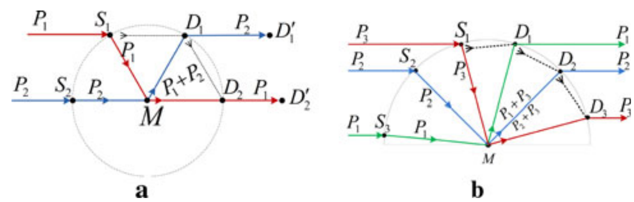

Fig. 3 The partial Star-NC schemes. **a**. Size 2, **b**. Size 3

Note that, in general, D_1 through D_n are not necessarily the final destination of P_1 through P_n. Instead, these nodes act as intermediate nodes, forwarding the packets to other nodes such as D'_1 through D'_n. In this paper, for simplicity, the nodes D'_1 through D'_n are not shown for some of the figures.

Precisely, the NC for this structure, herein referred to as Star-NC, consists of three steps:

1) Coding at input nodes: S_i $(1 \leq i \leq \mathrm{n})$ sends its encoded packets to the relay node.
2) Coding at relay node: M broadcasts its encoded packets (e.g. $P_1 + P_3$ in the above example) to output nodes.
3) Decoding at output nodes: D_i $(1 \leq i \leq \mathrm{n})$ decodes desired packet using both encoded packets received from M and the overheard packets from the neighbors (i.e., a subset of input/output nodes) and then forwards them to the next hops. For example, D_3 in Fig. 2(a), decodes its packet (P_2) by means of XORing $P_1 + P_2 + P_3$ (overheard from S_3) with the encoded packet $P_1 + P_3$ (received from M).

Note that, for some schemes such as Fig. 3(a), the first step is not needed since no overhearing is done by the input nodes. Thus Star-NC without/with coding at input nodes will be a *two/three-hop* coding scheme.

Each scenario of Star-NC scheme corresponds to a specific routing pattern of flows between input and output nodes. The routing pattern is identified by a unique-spanning mapping from the input nodes to the output nodes. We define *target permutation* of Star-NC as a permutation of

(1,2,…,n) that respectively identifies the indexes of the packets received by D_1 through D_n, e.g., (3,1,2) is the target permutation of the Star-NC scheme in Fig. 2(a). Generally, Star-NC only applies NC to the flows going through the paths identified by the target permutation. The packets of other flows are processed via regular routing. Obviously, for the n-input n-output star structure, there are n! distinct states for the target permutation. Note that each of them does not necessarily lead to a full Star-NC scheme of size n. Actually for some target permutation there is a direct link between some pairs of (input, output) nodes and thus the corresponding input node can send its packet directly to output node, i.e., no need to forward by the relay node. In this situation, the coding scheme converts to a partial-star NC of smaller size plus a set of direct transmissions between corresponding (input, output) nodes. From a practical point of view, this situation is not happened for multiple unicast sessions which are routed by an algorithm with shortest-path metric. For example, routing of three unicast sessions for target permutations (2,3,1), (3,2,1) and (2,3,1) corresponds to a full-star NC of size 3 (respectively shown in Fig. 2), whereas for (2,1,3)/(1,3,2), it leads to a partial-star NC of size 2 plus a direct transmission on link (S_1, D_i)/(S_3, D_3). The last permutation, i.e., (1,2,3), is a trivial case in which Star-NC has no benefit over traditional routing which is done by seven transmissions.

It is not necessary for Star-NC to have distinct elements for the set of input/output nodes, i.e., some of the nodes are the same. This means that the paths, correspond to multiple unicast sessions, are not necessarily *edge-disjoint*, i.e., there is an edge which lies in more than one path. Hence, a star structure of size n may be composed of less than 2n distinct nodes. For example, in Star-NC of Fig. 2(a), if D_2 overlaps D_1, we have a star structure of size 3 with 5 nodes. The same will be occurred if S_1 overlaps S_2 and has a position located in the transmission range of both D_1 and S_3. Thus, taking a specific Star-NC scheme as basis, we can generate various coding schemes of smaller size by overlapping some of the nodes.

2.2.1 Opportunistic listening in Star-NC

We can see that opportunistic listening has the crucial role in reducing the number of transmissions of the relay node. In fact, wireless networks exhibit significant data redundancy [24], i.e., there is a large overlap in the information available to the nodes. First, as a packet travels multiple hops, its content becomes known to many nodes. Further, wireless broadcast amplifies this redundancy because at each hop it delivers the same packet to multiple nodes within the transmitter's radio range. Network coding can take the advantage of data redundancy along with the broadcast nature of the wireless medium to reduce the number of transmissions. Therefore, the more flexible ways of overhearing an NC scheme has, the more opportunities for NC will be created.

Our scheme has unique features compared to COPE-type coding scheme from opportunistic listening view. Particularly, It benefits from the proximity of all nodes around the relay node, namely, the combinations (next-hop, previous-hop), (next-hop, next-hop) and (previous-hop, previous-hop) are legal for listener and overheard nodes base to the relay node. However, the opportunistic listening for COPE-Type NC (based on the Sengupta's formulation [15]) is only limited to (next-hop, previous-hop) combination, e.g., in Fig. 1(b) and (c). Precisely in addition to (next-hop, previous-hop), *two-hop* Star-NC has the case (next-hop, next-hop) while *three-hop* Star-NC has both the combinations (previous-hop, previous-hop) and (next-hop, next-hop) for opportunistic listening. These extra forms of opportunistic listening allow Star-NC to exploit the broadcast nature of wireless medium more efficient than COPE.

For example, the simple schemes in Fig. 3(a) and (b) benefit from the closeness of D_1 and D_2 with opportunistic listening. This feature identifies the Star-NC scheme from a COPE-type one. In fact, the overhearing of D_1 (by D_2) has a critical role in coding opportunity, i.e., if we limit the overhearing to COPE's pattern, no coding is applicable for these structures. Furthermore, employing the (previous-hop, previous-hop) overhearing allows an input node to send a coded packet to the relay node instead of a native one, thereby making more data redundancy. For instance in Fig. 2(a), opportunistic listening of both 'S_2 from S_1' and 'S_3 from S$_2$' brings the information of packets P_1 and P_2 to D_3.

On the other hand, opportunistic listening imposes a geographical restriction to the nodes of star structure where each node is constrained to be in the transmission range of the nodes that overhears them. For the schemes in Fig. 2, some nodes must be capable of overhearing neighbor nodes in both directions [e.g. D_2 in Fig. 2(b)] while some of the nodes need to overhear a neighbor in only one direction [e.g. D_1 in Fig. 2(a)]. Against, for partial Star-NC, as mentioned earlier, if a node overhears another node, the overhearing is restricted either to clockwise or counter clockwise direction.

2.2.2 Practical issues

It is worth noting that two-hop Star-NC (such as schemes in Fig. 3) does not require any coordination among the nodes since no coding is performed at input nodes. Therefore, like COPE, the relay node is the only node that generates coded packet and thus can make decision of NC with the received packets from the input nodes. In other words, the NC opportunities for node M are identified by its local information.

On the other hand, in three-hop Star-NC (such as schemes in Fig. 2), we need a type of coordination among the input nodes and the relay node. To follow the first step of coding, the input nodes must be able to detect the flows belonging to the desired unicast sessions identified by target permutation. It is clear that the implementation of flow coordination in a general wireless network is a more challenging task. Thus if the cost of flow coordination is not acceptable, we can limit the usage of Star-NC to two-hop coding scheme. Next, we will show that this group (two-hop schemes) is more practical in WMN and itself contains a large portion of Star-NC opportunities. However, we think that the flow coordination is practical for specific networks such as wireless mesh networks. Indeed a wireless mesh network has the following features:

1. The topology is relatively stable except for the occasional failure of the nodes or addition of a new node.
2. Traffic flow aggregated from a large number of end users changes occasionally, and hence could be assumed constant for an extended period of time, i.e., the set of unicast pairs remain stable over a large time-scale.

Thus an inter-flow coordination among a set of nodes is applicable for a large period of time. This allows for time to re-coordinate among the nodes when the flow structure changes. Flow coordination could be triggered by the relay node when finding a suitable Star-NC scheme. After that, since the input nodes must send their packets (likely coded) to the relay node, each node must be capable of detecting the packets that belong to desired flows of the Star-NC scheme. In the case of source routing protocol, it is simply accessible by header information of each packet. Otherwise it could be provided by means of the relay node's route table, i.e., the relay node sends its route table to the input nodes when it triggers a specific Star-NC scheme.

2.2.3 Theoretical challenges

The effectiveness of a Star-NC scheme can be explained either by the number of required transmissions or the number of opportunistic listening. Note that each target permutation in Star-NC has different coding in terms of the number of transmissions from the relay node and the overhearing pattern of the nodes around the relay node. The former has impact on the total number of transmissions in the network and accordingly on the aggregate throughput of the network. The latter imposes some constraints on the geographical position of the nodes. Thus a coding with less overhearing has the fewer limitations and correspondingly more opportunities of coding will be exploited throughout the network. It is an interesting problem to find the most effective NC for a given star structure and a desired target permutation. Answering this question requires a deep insight into the analysis of a general star structure which is subject of an independent work. In this paper, we restrict the use of Star-NC schemes to star structure of size 2 and 3. The NC schemes for these structures for both full and partial star are respectively depicted in Figs. 2 and 3.

3 Network coding in multi-channel system

Throughout this paper, we assume the set of unicast sessions remains stable over a large time-scale, and thus allowing for time to re-assign the channels to all interfaces when the flow structure changes. We further adopt a fixed channel assignment for a given traffic, i.e., the channels assigned to each interface on each node are fixed at the flow level and reassigned when the traffic model is changed. We mainly focus on the scenarios where nodes of the network are legacy 802.11a/b/g access points equipped with a diverse number of interfaces. Obviously, for utilizing the available radio resources, the number of channels must not be fewer than the maximum number of interfaces mounted on a node.

Network coding for multiple unicast sessions is based on two primitive operations; multicasting and opportunistic listening. The former is done at the output links of the relay node while the latter is taken by the nodes around the relay node. In a single channel system, no condition is required to perform these operations. But in multi-channel system, the feasibility of them completely depends on the state of channel assignment. To provide multicasting, a sufficient condition is that the outgoing links of the relay node are on the same channel frequency. This condition is a primary requirement for all NC paradigms. More specifically, for the information exchange paradigm, this is the only constraint of NC. However, for the opportunistic listening and hybrid paradigms, in addition to the above constraint, it is necessary for the listener node to have a radio tuned to the channel frequency of the overheard link. This constraint can be a rigid restriction for NC opportunities in a system with a large number of orthogonal channels since the neighboring nodes tend to use orthogonal frequency bands to reduce co-channel interference.

In summary, the constraints related to coding in multi-channel system are categorized in three main parts:

1) The constraints related to outgoing links of the relay node: These links must be on the same channel frequency to provide multicast transmission.

2) The constraints related to incoming links of the relay node: Opportunistic listening enforces a node to overhear the transmissions on a specific incoming link. For instance, the transmission on link (S_1, M) in the schemes of Fig. 2 and 3 is overheard by D_1. The same is true for link (S_1, M) and node D_2 in the COPE scheme shown in Fig. 1(b). This constraint imposes listener node to have an interface tuned to the channel frequency of the overheard link.
3) The constraints related to the links outside of the coding structure: This constraint appears when transmission on a link outside of the coding structure is overheard by an output node. For example, in Fig. 2(a), the transmission on link (D_1, D_2) is overheard by node D_2.

Note that type-1 constraint exists in every coding structure. Type-2 presents in coding structures based on opportunistic listening. Type-3 only appears in star structure coding. In spite of COPE-type NC which is based on either packet exchange or opportunistic listening, Star-NC is a coding scheme with opportunistic listening paradigm which has at least an (next-hop, next-hip) overhearing pattern. Thus we can see all of the mentioned constraint types in a single Star-NC scheme.

To our best knowledge, all of few work on NC in multi-channel system, including [19] and [16], only studied the information exchange paradigm. This is due to the simplicity of its mathematical formulation in the absence of opportunistic listening. Furthermore, the packet exchange method has more flexibility for the type of input traffic relative to opportunistic listening. In fact, the type of incoming packet to coding structure must be native (i.e., uncoded) in opportunistic listening based paradigms. However, in packet exchange the coding structure can accept coded traffic (in addition to native traffic) as input, i.e., input traffic is coded by another coding structure [15].

4 Star-NC optimization framework for general network topologies

In this section, we formulate a linear programming (LP) framework to find the maximum throughput of the network using Star-NC scheme in a multi-channel system. The framework uses an LP technique similar to ones used in [8–10]. The difference is that our scheme is based on opportunistic listening that is missing in [19] and [16]. They only consider the information exchange paradigm of COPE-type NC in a simple Alice-Bob topology. Moreover, our framework significantly differs from [15] from two aspects; first, it is limited to the formulation of NC in single channel system, and the second it has a two-hop coding scheme with opportunistic listening (COPE-type NC). However, our formulation covers multi-channel system along with the star structure network coding (Star-NC) which is a two/three-hop coding scheme.

4.1 Notations and modeling assumptions

The notations and modeling assumptions are listed in Table 1. Note that, for simple representation of the LP in multi-channel system, we use a trick that includes the channel frequency in the link definition, i.e., we define a link as a triple (u, v, c) which denotes a link on channel c from node u to node v. This modeling enforces the variables such as $E^C(e)$, which denotes the set of links conflicting by e, to hide any mark about channel frequency while implicitly considers only the links that have the same channel frequency with link e.

We assume the network connectivity to be symmetric, i.e., link $e = (u, v, c) \in E$ if and only if $\bar{e} = (v, u, c) \in E$. Furthermore, we use the protocol model of interference introduced by Gupta and Kumar [25], i.e., two nodes have a link if their distance is less than *communication range* and are interfered if their distance is less than *interference range*. Also, two links $e_1 = (i_1, j_1, c)$ and $e_2 = (i_2, j_2, c)$ are interfered either as j_1 is within the interference range of i_2 or j_2 is within the interference range of i_1.

4.2 Star-NC modeling

We present Star-NC by a five tuple $(M, \mathcal{S}, \mathcal{D}, \pi, n)$ and denote it as ξ, where M is the relay node, π is the target permutation for routing and n denotes the size of star ($n = |\mathcal{S}| = |\mathcal{D}|$). Further, $\mathcal{S}$ and $\mathcal{D}$, respectively, represent the incoming and outgoing links of node M. We refer to component x of ξ as $x(\xi)$, e.g. $M(\xi), \mathcal{S}(\xi), \mathcal{D}(\xi)$ and $n(\xi)$. Further, $N(\xi)$ denotes the number of packets that are sent by node M during a coding operation. In this modeling the following conditions must be satisfied:

1. A set of n unicast transmissions, intersecting each other at M, are to be routed from input nodes to output nodes according to target permutation π.
2. A set of input/output nodes must be located on the star structure in order that for the intended coding:
   a. Each node is constrained to be in the transmission range of the nodes that overhears their transmissions.
   b. Each node must satisfy the channel assignment conditions for multi-channel system, i.e., described in Sect. 2.

For example, we bring the channel assignment constraints for some schemes of Figs. 1, 2 and 3 in Table 2.

Table 1 Notations and variables list used in system modeling

G	The network topology
V	The set of node of the network. For the network G, it is denoted by $V[G]$ too
E	The set of directed links in the network. For the network G, it is denoted by $E[G]$ too
E^{+}(v)	The set of incoming links incident on node v
E^{-}(v)	The set of outgoing links incident on node v
D	The set of traffic demands (related to unicast sessions)
ω	The set of no-overlapping channel frequencies
$e = (u, v, c)$	The directed link from u to v on channel c
$Cap(e)$	Capacity of data link e
N_u	The number of interfaces of node u
Q_u^i	The i-th interface of node u
$Adj\ (u)$	The nodes of the networks which are in the transmission range of node u
$C(u)$	The set of channel frequencies which used by the node u
$Ch(e)$	The channel frequency of link e
$D(k)$	Traffic amount which requested by session k
λ_k	The end-to-end throughput for the demand k, namely, a portion of $D(k)$ which can be routed by the network
$s(k)$	Source node of traffic demand k
$d(k)$	Destination node of traffic demand k
$\mathcal{S}$	Input links of star structure from input nodes to relay node, namely, $\mathcal{S} = \{S_1, \ldots, S_n\} \times \{M\}$
$\mathcal{D}$	Output links of star structure from relay node to output nodes, namely, $\mathcal{D} = \{M\} \times \{D_1, \ldots, D_n\}$
$\xi = (M, \mathcal{S}, \mathcal{D}, \pi, n)$	Star structure of size n by five tuples: M is the relay node, $\mathcal{S}/\mathcal{D}$ denotes the input/output links to/from M, π is target permutation for routing ($n = \lvert\mathcal{S}\rvert = \lvert\mathcal{D}\rvert$).
$N(\xi)$	The number of required (coded/uncoded) packets to be sent from the relay node of ξ to output nodes.
Γ	Set of all star structures which employ Star-NC scheme
$f(e)$	Total flow rate passed through link e
$F_k(P)$	The flow rate of demand k over path P
P_k	The set of available paths for demand k
$z_e^k(P)$	The portion of the traffic on path P for demand k that is transmitted as uncoded from link e
$f^{NC}(\xi)$	Flow rate of coded traffic for star ξ which broadcast by M to output nodes
$f_v\left(e, e^{'}\right)$	Total flow rate which enters to node v from incoming link e and exit it from outgoing link $e^{'}$
$E^C(e)$	The set links which conflict by link e. These links have the same channel as the node e
$\Gamma^C(e)$	The set of star structures which conflict by link e. This holds when e remains in conflict with any output links of ξ
$Int(e)$	The interference intensity of the link e, which denoted by left hand side of constraint (9)

We can see that the information exchange paradigm has only the primary constraint about multicasting. However, the other paradigms have one/two constraint(s) about the listener nodes, in addition to the primary constraint.

We use a straightforward method to generate all the valid Star-NC opportunities for a given network. First, by routing the desired traffic model, we can find all the unicast sessions passed through each node M. Let $(e, e^{'})$ denotes two consecutive links referred as a *link-pair*. Assume $f_M(e, e^{'})$ denotes the traffic rate of the link-pair $(e, e^{'})$ which enters to node M from incoming link e and exits from outgoing link $e^{'}$. Consequently, $f_M(e, e^{'})$consists of the flows that respectively pass through e and $e^{'}$. We can see that the number of non-zero $f_M(e, e^{'})$is upper bounded by $d_M(d_M - 1)$ where d_M is the degree of node M ($d_M = d_M^+ = d_M^-$). By this definition, the number of non-zero flows at node M depends only on d_M, and neither on the number of traffic demands nor on the number of flows in the network. Since an opportunity of Star-NC at node M of size n is created by appropriate selection of n different flows passed through M, the number of distinct Star-NC schemes of size n, for a giver network $G(V, E)$ is bounded by:

$$\sum_{M\in V}\binom{d_M(d_M-1)}{n} \le \sum_{M\in V}\binom{d_M^2}{n} \le \sum_{M\in V}\frac{1}{n!}d_M^{2n} \le \frac{1}{n!}\left(\sum_{M\in V} d_M\right)^{2n} = \frac{4^n}{n!}|E|^{2n} \tag{1}$$

This means that the number of star structures of size n is $O(|E|^{2n})$. Since the nodes in a wireless mesh network have small degree, the size of star (i.e. n) would become relatively small. More specifically, as we only consider the star structure of size 2 and 3 for the evaluation, the number of Star-NC opportunities will be $O\left(|E|^4 + |E|^6\right)$. Let Γ denotes the set of all valid Star-NC opportunities. Hence, it is relatively simple and fast to generate Γ for a given network with a set of traffic demands.

4.3 LP formulation

We model the traffic as a set of traffic demand denoted by D. The demand k corresponds to $D(K)$ amount of traffic (e.g. in Mbps) requested by source node $s(k)$ to be routed to destination node $d(k)$. As in [8–10], we define the throughput as a multiplier λ such that for each demand k, at least $\lambda D(k)$ amount of requested traffic is guaranteed to be routed by the network. This definition holds the system as linear while provides a means for fairness. Note that the aggregated network throughput is equal to the sum of all

Table 2 The requirements for some Star/COPE-type NC schemes in multi-channel system

	Star-NC		COPE		
	Partial n = 2	Full n = 3, π = (3,1,2)	Structure (a)	Structure (b)	Structure (c)
Outgoing link from M	Ch(MS$_1$) = Ch(MS$_2$)	Ch(MS$_1$) = Ch(MS$_2$) = Ch(MS$_3$)	Ch(MA) = Ch(MB)	Ch(MD$_1$) = Ch(MD$_2$)	Ch(MA) = Ch(MD)
Incoming links to M	Ch(S$_1$M) $\in C(D_1)$	Ch(S$_1$M) $\in C(S_2)$ Ch(S$_2$M) $\in C(S_3)$ Ch(S$_1$M) $\in C(D_1)$ Ch(S$_3$M) $\in C(D_3)$	–	Ch(S$_1$M) $\in C(D_2)$ Ch(S$_2$M) $\in C(D_1)$	Ch(SM) $\in C(D)$
Links outside of Structure	Ch($D_1 D_1^{'}$)$\in C(D_2)$	Ch($D_1 D_1^{'}$)$\in C(D_2)$	–	–	–

routed traffic for each demand, i.e., $\sum_{k\in D} D(k)\lambda$. We have the following set of constraints.

Fairness constraint In our system, we consider multi-path routing. Let P_k be the set of available paths to route demand k from $s(k)$ to $d(k)$. Assume $F_k(P)$ denotes the amount of traffic on path P for routing demand k, where $P \in P_k$. Thus, the total traffic routed for demand k equals to $\sum_{P\in P_k} F_k(P)$. On the other hand, this amount of traffic must be equal to the requested traffic for demand k multiplied by the throughput, i.e., $D(k)\lambda$. This is stated in constraint (2).

Coding constraint For this constraint, we need to know the amount of traffic for the flows intersecting each other at $M(\xi)$. It is necessary that the traffic on incoming link of a star structure, which participates in coding, received as native, i.e., it is not coded by another star structure. To derive this condition, we use $z_e^k(P)$ to denote be the portion of the traffic on path P for demand k that is transmitted via link e as native. Thus for each combination of incoming link e_1 and outgoing link e_2 at node M, the portion of routed traffic, that received as native, is equal to $\sum_{k\in D}\sum_{P\in P_k:P\ni e_1 e_2} z_{e_1}^k(P)$. Since the opportunity of Star-NC arises when the relay node has a set of n packets in its queue, each of them received from one of the input nodes, the rate of coding at relay node is the minimum rate of incoming links. Further, as the relay node generates $N_\pi(n)$ packets (likely coded) for each group of n collected packets, for each combination of incoming/outgoing link $e_1 e_2$ in a specific star structure ξ, we must have:

$$\frac{f^{NC}(\xi)}{N(\xi)} \le \sum_{k\in D}\sum_{P\in P_k:P\ni e_1 e_2} z_{e_1}^k(P)$$

Here, $f^{NC}(\xi)$ denotes the rate by which $M(\xi)$ generates the coded packets and broadcasts to the output nodes. Constraint (6) is the extension of the above constraint since the pair $e_1 e_2$ may participate in more than one Star-NC schemes. We can write a balance constraint for $F_k(P)$ in terms of $z_e^k(P)$ and $f^{NC}(\xi)$ in (7). The total transmitted traffic on link-pair (e_1, e_2), i.e., entering to e_1 and exiting through e_2, appears on left hand side (LHS) of the equation. The first portion on right hand side (RHS), is the amount of routed traffic that participates in coding while the second portion is the amount of routed traffic that transmits as native, i.e., does not participate in any coding. Furthermore, $z_e^k(P)$ is bounded by constraints (3) and (4).

Traffic splitting constraint Let $f(e)$ be the total flow rate of the traffic on link e. Obviously, this flow for the nodes outside of any star structure is only the unicast traffic. Further, as every input node of the star structure, in each transmission, sends its packet (likely XOR-ed with the overheard packets) and does not generate a coded packet without including its packet, we also consider the traffic of input links as unicast type. On the other hand, the output links of a star structure are the only links that transmit *multicast traffic* in addition to the unicast traffic. This traffic is denoted by $f^{NC}(\xi)$ and interchangeably referred as NC traffic. It is only generated by the relay node and always broadcasted to all output nodes, i.e., $f^{NC}(\xi)$ belongs to every output link of ξ. Thus, by the above assumption, $M(\xi)$ is the only node among the nodes of ξ that transmits traffic of both unicast and multicast type. Note that $f^{NC}(\xi)$ is the rate of NC traffic which is created by mixing the information of all n incoming flows to ξ. As mentioned above, relay node generates $N(\xi)$ coded-packets for each group of n collected packets. Thus the rate that each output node decodes it information is equal to $\frac{f^{NC}(\xi)}{N(\xi)}$. We can assume that the amount of NC traffic related to each flow of ξ is equal to $\frac{f^{NC}(\xi)}{N(\xi)}$, too. Suppose that ξ is the only star structure that includes has e as output link. Then $f(e)$ in (8) is the total traffic on link e, out of which the portion $\frac{f^{NC}(\xi)}{N(\xi)}$ is the NC traffic and the rest (i.e., $f(e) - \frac{f^{NC}(\xi)}{N(\xi)}$) is unicast one. If link e belongs to more than one star structure, the term $\frac{f^{NC}(\xi)}{N(\xi)}$ is replaced by $\sum_{\forall \xi, \mathcal{D}(\xi) \ni e} \frac{f^{NC}(\xi)}{N(\xi)}$. We use this traffic classification to represent the interference constraint.

Maximize λ Subject to

$$\sum_{P \in P_k} F_k(P) = D(k)\lambda \quad \forall k \in D \tag{2}$$

$$z_e^k(P) \le F_k(P) \quad \forall k \in D,\ P \in P_k : P \ni e \tag{3}$$

$$z_e^k(P) = F_k(P) \quad \forall k \in D, P \in P_k : P \ni e, t(e) = s(k) \tag{4}$$

$$\mathcal{SD}(\xi) = \{(e_1, e_2) | e_1 = \mathcal{S}(\xi)_i, e_2 = \mathcal{D}(\xi)_{\pi(i)} \colon 1 \le i \le n(\xi)\} \tag{5}$$

$$\sum_{\xi \in \Gamma : (e_1, e_2) \in \mathcal{SD}(\xi)} \frac{f^{NC}(\xi)}{N(\xi)} \le \sum_{k \in D} \sum_{P \in P_k : P \ni e_1 e_2} z_{e_1}^k(P) \quad \forall M \in V, e_1 \in E^-(M), e_2 \in E^+(M) \tag{6}$$

$$\sum_{k \in D} \sum_{P \in P_k : P \ni e_1 e_2} F_k(P) = \sum_{\xi \in \Gamma : (e_1, e_2) \in \mathcal{SD}(\xi)} \frac{f^{NC}(\xi)}{N(\xi)} + \sum_{k \in D} \sum_{P \in P_k : P \ni e_1 e_2} z_{e_2}^k(P) \quad \forall M \in V, e_1 \in E^-(M), e_2 \in E^+(M) \tag{7}$$

$$f(e) = \sum_{k \in D} \sum_{P \in P_k : P \ni e} F_k(P) \quad \forall e \in E \tag{8}$$

$$\sum_{\substack{e' \in E^C(e), \\ \nexists \xi \in \Gamma^C(e) : e' \in \mathcal{D}(\xi)}} f(e') + \sum_{\xi \in \Gamma^C(e)} f^{NC}(\xi) + \sum_{\substack{e' \in E^C(e), \\ \exists \xi \in \Gamma^C(e) : e' \in \mathcal{D}(\xi)}} \left(f(e') - \sum_{\forall \xi \in \Gamma^C(e), \mathcal{D}(\xi) \ni e'} \frac{f^{NC}(\xi)}{N(\xi)} \right) \le C(e) \quad \forall e \in E \tag{9}$$

$$0 \le f(e) \le C(e) \quad \forall e \in E \tag{10}$$

$$0 \le f^{NC}(\xi) \le \min_{e \in \mathcal{S}(\xi) \cup \mathcal{D}(\xi)} C(e) \quad \forall \xi \in \Gamma \tag{11}$$

$$\sum_{e \in E^+(v)} f(e) - \sum_{e \in E^-(v)} f(e) = \begin{cases} 0 & \forall v \ne s(k), d(k), k \in D \\ \sum_{k \in D, v = s(k)} \lambda_k - \sum_{k \in D, v = d(k)} \lambda_k & \forall v = s(k), d(k), k \in D \end{cases} \tag{12}$$

Interference constraint We consider three different link and traffic combinations : (1) the link which is not the output link of any star structure and thus transmits only unicast traffic, (2) the output link of star structure with NC traffic and (3) the output link of star structure with unicast traffic. This classification identifies three types of interference for each link e. Each type is corresponded to a term in LHS of constraint (9). In the first term, we seek the set of links such as e' which is not the output link of any star structure. In the second, we explore the set of star structures such as ξ whose NC traffic conflicts with the transmissions on link e. This holds if e is interfered with any of the output links of ξ. In the last term, we look for the output link e' of any ξ which interferes with link e while transmitting unicast traffic. As mentioned above, the amount of unicast traffic over e' is equal to the total traffic going through e' minus the rate of NC flow decoded at the output node of e', i.e., $f(e') - \sum_{\forall \xi, \mathcal{D}(\xi) \ni e'} \frac{f^{NC}(\xi)}{N(\xi)}$.

Routing constraint The constraint given by (12) maintains the flow conservation at every node of the network. For a relay node which is neither source nor destination of any session, the difference of incoming and outgoing traffic is zero. For other node which is either sink or source or both, the difference becomes a non-zero value determined by RHS of constraint (12).

Link capacity constraint: Equations (10) and (11) limit the flow rate of a link to its capacity, respectively for unicast and NC traffic.

The above constraints form our linear programming setup for maximizing λ. The complete LP formulation is shown in (2) through (12). It is worth noting that the LP represents the general form of throughput optimization for non-NC scheme, in addition to Star-NC scheme. It is enough to set $\Gamma = \emptyset$ to have a throughput optimization for the non-NC scheme. Further, the coding opportunities created by other coding scheme, such as COPE, can be incorporated to the LP.

5 Coding-aware channel assignment

In this section we propose a channel assignment algorithm using an extension of *simulated annealing* method [21]. Simulated annealing is a popular local search meta-heuristic used to address discrete optimization problems. The key feature is that it provides a means for escaping local optima by allowing hill-climbing moves (i.e., moves which worsen the objective function value) in hopes of finding a global optimum.

In simulated annealing, each feasible set of variables is called a *solution*. For channel assignment problem, a solution mainly reflects the state of channel frequencies used in each node. The algorithm, in first step, creates an initial solution, and then iteratively updates the solution by producing neighboring solutions based on the current solution. To solve a combinatorial optimization problem using simulated annealing, the key step is to design appropriate *initialization* and *update* algorithms.

5.1 Initialization step

We assume single-link connectivity, i.e., there is only a single link between two adjacent nodes if they share some channels. The initial situation is generated using Algorithm 1 (Initialize-Channel-Assignment). First, the algorithm for each node selects randomly a set of non-overlapping channels whose size is equal to the number of interfaces. Then, these channels are respectively assigned to node's interfaces. In second step, the subroutine Link-Assign is called for each node to establish a channel with neighbor nodes. For each neighbor pair (u, v), first, the set of common channels between u and v was specified, one of them is randomly picked and assigned to the links between u and v. Note that, with the assumption of symmetric connectivity, both links (u, v) and (u, v) use the same channel frequency.

by $I_{\mathcal{H}_c}$, which are interfered with all elements in $\mathcal{H}_c$. The main idea behind $I_{\mathcal{H}_c}$ is to find a proper link, among the candidate links, in which changing its channel frequency eliminates the saturation state of the links in $\mathcal{H}_c$. For the case that $I_{\mathcal{H}_c}$ is an empty set (due to the large number of links in $\mathcal{H}_c$), we randomly remove an elements of $\mathcal{H}_c$ and recalculate $I_{\mathcal{H}_c}$.

Following the current solution, neighboring solutions are generated subject to elements of $I_{\mathcal{H}_c}$. Let $\mathcal{C}$ denotes the current solution. For each $e = (u, v, c) \in I_{\mathcal{H}_c}$, we can generate an new solution $\mathcal{C}_{\mathrm{N}}$ from $\mathcal{C}$ by switching the channel frequency of e from c to a new $c' \in \omega$. This is done by injecting channel c' into interfaces of both u and v, i.e., if each of them has not an interface tuned to c', the channel frequency of the interface tuned to c is replaced by c'. After that, we call the Link-Assign subroutine for both u and v to update their links with the neighbors. We take $\mathcal{C}_{\mathrm{N}}$ as a neighbor solution if the resulting graph is connected. Note that the number of neighbor solutions, created in this manner, is upper bounded by $\sum_{c\in\omega} |I_{\mathcal{H}_c}| \cdot (|\omega| - 1)$.

The complete process of channel assignment is detailed in Algorithm 4 (Find-Optimized-Channel-Assignment). As

Algorithm 1. Initialize channel assignment

```
Link-Assign(u : Node)
    Clear e ∈ E[G] which u is one of its endpoints
    for each v ∈ Adj(u) do
        select randomly c ∈ C(u)∩C(u)
        create e = (u, v, c) and ē = (v, u, c) and add them into E[G]
Initialize-Channel-Assignment(G)
    for each u ∈ V[G] do
        Select a random set of distinct N_u channel (c_1, c_2, .. c_{N_u}) from ω
        and assign it to (Q_u^1, Q_u^2, .. Q_u^{N_u})
    for each u ∈ V[G] do
        Link-Assign(G, u)
```

5.2 Update step

First, we describe Algorithm 2 which is employed to generate a set of neighboring solutions for the current solution. Assume $\mathcal{H}_c$ denotes the set of links on channel c which saturated in the current solution, i.e., the interference of them [i.e., LHS of constraint (9)] reaches to the link capacity. We can simply find $\mathcal{H}_c$ for each $c \in \omega$. After that, for each $\mathcal{H}_c$, we try to find a set of candidate links, denoted

mentioned above, we use an extended method of simulated annealing to search the state space of the problem, thereby reaching to a near global optimum. First, we generate an initial solution $\mathcal{C}$ using the algorithm Init-Channel-Assignment. Next, we begin a loop by finding $\mathcal{S}_{\mathcal{C}}$, the set of neighbor solutions of $\mathcal{C}$, and computing the corresponding throughput using Algorithm 3. We further select an element of $\mathcal{S}_{\mathcal{C}}$ as a candidate for next solution in which has the maximum throughput among the elements of $\mathcal{S}_{\mathcal{C}}$ (Algorithm 2).

Algorithm 2. Finding neighbor solutions of the current solution

```
Input   : 𝒞 : Current Channel Assignment
Output : 𝒮_𝒞 : Set of Neighbor Solutions
Find-Neighbor-Solutions(𝒞)
𝒮_𝒞 ⟵ ∅
1.      For each c ∈ ω Find
                ℋ_c = {e ∈ E | Ch(e) = c and Int(e) = Cap(e)}
2.      For each c ∈ ω , ℋ_c ≠ ∅ do
    3. Find I_ℋc = {e ∈ E | ∀é ∈ ℋ_c : e ∈ E^c(é)}
    4. Select two links of I_ℋc with the maximum load and delete the others
    5. For each e = (u, v, c) ∈ I_ℋc
          For each ć ∈ ω , ć ≠ c
              //Create 𝒞_N from 𝒞 by replacing c by ć in e
              𝒞_N ⟵ 𝒞
              If (ć ∉ C(u)∩C(u))
                  If (u has  not an interface on channel ć)
                      Assign ć to the interface of u tuned to c
                      In 𝒞_N do Link-Assign(u)
                  If (v has  not an interface on channel ć)
                      Assign ć to the interface of v tuned to c
                      In 𝒞_N do Link-Assign(v)
              end if
              Tune the (u, v) and (v, u) of  to channel ć
              if (𝒞_N correspond to a connected graph)
                      Add 𝒞_N to 𝒮_𝒞
return 𝒮_𝒞
```

Algorithm 3. Finding Throughput

```
Inputs   : G : Network Topology,  𝓒 : Channel assignment
           R  : Routing method,     D  : Set of traffic demands
Output   : Aggregated throughput of the network
Find-Throughput (G, 𝓒, R, D)
1.    Assign channels to G according to configuration 𝓒
2.    For each demand k ∈ D
          if (R is single-path)
                  Find the shortest path from s(k) to d(k) and add it to P_K
          else if (R is multi-path)
                  Find all the edge-disjoint paths from s(k) to d(k) and add it to P_K
          end if
3.    For each link  e ∈ E
          Compute f(e) for each e ∈ E in terms of F_k(P), P ∈ P_K
4.    For each node  M ∈ V
          S_M⟵{(e, é) | e ∈ E⁻(M), é ∈ E⁺(v): f_M(e, é) ≠ 0}
          For each S_n ⊆ S_M of size n
                  if (S_n creates an opportunity of Star-NC)
                          generate ξ from S_n and add it to Γ
                  end if
5.    Create the LP formulation using the route information and Γ
          Solve the LP using CPLEX to find λ
6.    if R is multi-path routing and ∃ P ∈ P_K, k ∈ D :  F_k(P) = 0
          For each demand k ∈ D
                  For each P ∈ P_K :  F_k(P) = 0
                          Remove P from P_K
          Go to step 3.
      end if
return ∑_{k∈D} D(k)λ
```

Algorithm 4. Finding optimized channel assignment.

```
Inputs  : G : Network topology, D : Traffic Demands, R : Routing method,
Output : C_max : Optimized channel assignment
Find-Optimized-Channel-Assignment(G, D, R)
        C ← Init-Channel-Assignment(G);
        C_max ← C
        τ ← τ_0
        L←1
        do {
                S_C ← Find-Neighbor-Solutions(C);
                For each C_x ∈ S_C
                        λ(C_x) ← Find-Throughput(G, C_x, R, D)
                Sort S_C in non-increasing order relative to λ(C_x)
                C_N ← Select a random solution from the first L elements of S_C
                if λ(C_N) > λ(C)
                        C ← C_N
                        C_max = Max(C, C_max)
                else if λ(C_N) < λ(C)
                        if e^((λ(C_N)- λ(C))/τ) >= rand[0,1)
                                C ← C_N
                L ← L + 1;
                if (C = C_N)
                        L ← 1;
                τ ← τ.θ
        } while (τ ≥ τ_F);
        return C_max
```

The theory of simulated annealing is based on the Metropolis acceptance criterion which models how a thermodynamic system moves from the current solution (state) to a candidate solution in which the energy content is being minimized. In our problem, the candidate solution $\mathcal{C}_{\mathrm{N}}$ is accepted as the next solution with the following probability, namely, *acceptance probability*:

$$\Pr[\text{Accept}\,\mathcal{C}_N\,\text{as next solution}] = \begin{cases} \exp\left(\frac{\lambda(\mathcal{C}_N)-\lambda(\mathcal{C})}{\tau}\right) & \lambda(\mathcal{C}_N)<\lambda(\mathcal{C}) \\ 1 & \lambda(\mathcal{C}_N)>\lambda(\mathcal{C}) \end{cases} \tag{13}$$

Where $\lambda(\mathcal{C})$ denotes the aggregate network throughput corresponds to $\mathcal{C}$. In summary, we replace $\mathcal{C}$ by $\mathcal{C}_{\mathrm{N}}$ and repeat the annealing loop if the throughput of $\mathcal{C}_{\mathrm{N}}$is better than $\mathcal{C}$. Otherwise, $\mathcal{C}$ is conditionally replaced by $\mathcal{C}_{\mathrm{N}}$with the probability equals to $\exp\left(\frac{\lambda(\mathcal{C}_{\mathrm{N}})-\lambda(\mathcal{C})}{\tau}\right)$. Here, τ is a control parameter called the *temperature parameter* of the loop iteration. This parameter is initialized to τ_0, a value larger than the expected λ, and multiply-decreased in each iteration, so that the acceptance probability approximates zero as the annealing algorithm approximates the optimal solution. Note that, in addition to τ, two other control parameters, θ and τ_{F}, are involved in the process. The first is the multiply-decreasing factor used to update the temperature τ. The suitable value for θ is between 0.7 and 1. The second, τ_{F}, is the minimum temperature that specifies the end of the loop iteration.

Note that the selection of next candidate solution from $\mathcal{S}_{\mathcal{C}}$ has a sliding-window mechanism, i.e., next solution is randomly selected among the L solutions with highest throughput. Here, L is the size of window which reset to 1 whenever the next solution is accepted; otherwise it is increased by one. Our method has slight differences with the basic algorithm of simulated annealing which can be summarized as:

1. The final solution of our algorithm, instead of the last accepted solution, is the one whose throughput is the maximum.
2. We ignore the case $\lambda(\mathcal{C}_N) = \lambda(\mathcal{C})$ which in basic algorithm has the acceptance probability equals to 1. This is due to the generation of a restricted loop which makes the algorithm fluctuating between a set of throughput-equal solutions.
3. We have a sliding-window mechanism to select the next candidate solution among the feasible candidates. Experimentally, we found that this method, like as case 2, prevents the algorithm to fall in a trap where it oscillates between a confined set of solutions.

It is worth mentioning that the simulated annealing is further used in the work of Zhang [19] for finding the optimum channel assignment. The main distinction of our method with [19], in addition to aforementioned differences, is that we adopt a gradual movement from the initial solution to near optimal solution, i.e., the algorithm, in each update step, only changes the channel frequency of a single link that affects the load of saturated links. This leads to a partial change in the channel state of the nodes in each update step, so that only affects on the interfered links of the candidate link. However, the changing in [19] has a sharp method, i.e., all of the links are forced to reassign channel according to an objective value named as *utilization ratio*. At the next section, we will compare our channel assignment algorithm with the work of Zhang.

6 Evaluation

In this section, we compute the performance of the above joint routing, NC and channel assignment over the non-NC scheme. First, we introduce the configurations including the network topology, traffic models, coding schemes and routing strategies. Next, we introduce our self-designed testbed tool for evaluation. Finally, the evaluation results are presented.

6.1 System configurations

6.1.1 Network topology

The target topology used for evaluation is shown in Fig. 4. The network consists of 49 nodes in a square of side 7 × 7 units where the position of the nodes was chosen randomly while maintaining connectivity. We assume that the transmission and interference range are equal to 1.7 and 2 unit, respectively. The average node degree is equal to 6.0. Note that Fig. 4 depicts the basic network topology in single channel configuration. In multi-channel system, the actual topology is identified after the assignment of channels to each node's interfaces. A link in the basic topology appears in corresponding multi-channel topology if both endpoints share a channel frequency. As a consequence, the set of links in multi-channel system is always a subset of the links in basic topology. We assume that the nodes in the network are equipped with a fixed number of interfaces. Moreover, number of non-overlapping channels in the network varies between 2 and 12 based on the desired evaluation.

6.1.2 Traffic models

We focus on two different randomly chosen traffic models, namely, *random* and *directional*. For the random model, as its name implies, the source and destination of each session

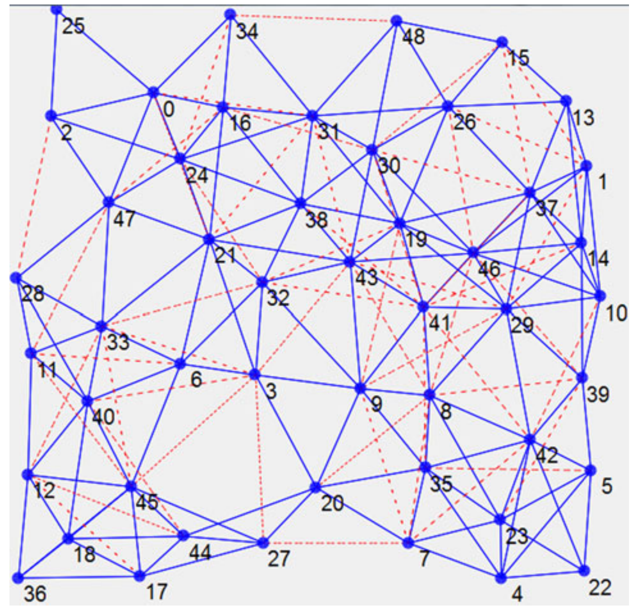

Fig. 4 The mesh network topology used for evaluation consisting of 49 nodes randomly located in a square of size 7 × 7. The communication/interference range are equals to 1.7 and 2. The *solid/dash lines* illusttrate the node-pairs which are in communication/interference range of each other

are chosen randomly from the network nodes. However, for directional model, the nodes are partitioned into two groups named as *senders* and *receivers*. For each session, the source and destination nodes are picked up from the sender and receiver groups, respectively. We use this model since it suits for wireless mesh networks that provide Internet connectivity to end users. For both models, we vary the number of demands from 50 to 500.

6.1.3 Routing strategies

We consider three routing strategies: (1) single-path routing (SP), (2) optimized single path routing (OSP) and (3) multi-path routing (MP). In particular, the single-path routing can be obtained by means of Dijkstra's algorithm and a metric which is to be minimized such as the Hop-count, joint Hop-count and physical distance or ETX metric [26]. The routing in multi-channel system has an important factor, in addition to delay metric, which influences on the network throughput. This factor is the diversity of channels among the chosen paths which can boost the capacity of multi-channel wireless networks. As mentioned above, WCETT is a source routing protocol with the awareness of both delay and channel diversity of the path. The tradeoff between delay and channel diversity is controlled by a tunable parameter referred as β. We select WCETT as our routing protocol with $\beta = 0.5$, i.e., the channel diversity is as important as delay in selecting the paths. Note that single-path routing itself is neither coding-aware nor interference-aware, i.e., the paths are selected based on the desired metric without considering either the coding opportunities or interference among the flows. To overcome these shortcomings, we are lead to examine multi-path routing.

As mentioned above, our LP formulation supports both the multi-path and single-path routing strategies. Multi-path routing considers *interference-aware* routing, i.e., the paths which minimize the interference among the flows are selected. Our algorithm for multi-path routing is based on the idea of *internally pairwise edge-disjoint paths* [27]. That is, for two nodes s and d, we first find the shortest path between s and d. Next, we remove the links belonging to this path and then explore the possible shortest path among the remaining links. We repeat the procedure until no route exists between s and d. At the end, we remove the possible cycles, in the presence of all of the links, from the paths found between s and d. The number of the paths that found by this manner depends on the *edge-connectivity* of the desired nodes in the graph, which is equal to the *edge-cut* of s and d [27]. For example, for a grid network, this number varies between 2 and 4 depending on the position of the source and destination nodes. From practical point of view, multi-path routing may not be applicable in most networks due to the high routing maintenance overhead. Thus, similar to [16], we consider *optimized single-path* routing in addition to single/multi-path routing. The key idea is to choose the path which provides the maximum flow among the routes specified by multi-path routing. The network throughput for optimized single-path routing is computed in two steps. First, we solve the LP formulation for multi-path routing, and then for each session, select the path that achieves the highest flow, namely, $P_{Opt} = Max_{P \in P_k} F_k(P)$. Second, we solve the LP again using the specified paths.

Note that optimized single-path routing, like multi-path routing, tries to minimize the interference among the flows, i.e., interference-aware routing. On the other hand, employing the NC scheme with multi-path or optimized single-path routing allows for another option in throughput optimization. As a result, the routing protocol tries to transmit traffic from the paths which create more coding opportunities, i.e., *coding-aware* routing [15], in addition to the paths that minimize interference among the flows. Thus, for multi-path and optimized single-path routing, both the notion of coding-aware and interference-aware routing is considered. Intuitively, multi-path routing can provide more coding opportunities than other routing methods. There is a fundamental tradeoff between the opposite effects of increased coding and increased interference to maximize network throughput. Our LP formulation provides a systematic approach for finding the routes that optimize the tradeoff and identifies the best routing choices.

6.1.4 Coding strategies

Our evaluation covers three main NC schemes (1) Star-NC (2) the COPE-type NC (3) the join Star and COPE-type NC scheme which are referred to as NC(STAR), NC(COPE) and NC(STAR + COPE) in the plots, respectively. The evaluation for COPE-type NC scheme is based on the LP formulations in [15]. For join Star and COPE, we extend our LP formulation to covers both Star and COPE-type NC schemes.

In our evaluation, we restrict Star-NC to the star structures of size 2 and 3. The size of Star-NC opportunities which are created around a specific node is dependent on its degree. Since the nodes in wireless mesh networks have small degrees (i.e., less than 8), the size of star structure is relatively small. In fact, the star structures of size 2 and 3 are the more often cases in wireless mesh networks. However, our LP formulation takes the general case into account. Note that the pattern which leads to a full-star structure of size 2 is the same as the COPE "X" topology (Fig. 1(b)). Therefore, for size 2 stars, we only consider partial ones (Fig. 2(b)), and accordingly in our evaluations, we ignore the opportunity of this scheme as a Star-NC and take it as the COPE-type NC. However, for size 3 stars we are engaged in both partial and full star-structures.

6.2 Putting it all together

We developed a testbed tool which integrates all of the above modeling options. Our evaluation testbed generates the LP system for a specific channel assignment along with any configuration of network topology, traffic model, NC scheme and routing strategy. We solve this LP using AMPL [28] with the CPLEX solver [29] to obtain the theoretically optimized throughput and the corresponding flows for the non-NC, Star-NC, COPE-type NC and joint Star and COPE NC schemes, respectively. The complete process is described in Algorithm 3. In most often cases for multi-path routing, it is necessary to solve the LP formulation more than one times since the load of some links becomes zero while the throughput evaluation is taken by considering the interference constraints correspond to these links. Thus, after each LP solution for multi-path routing, our testbed tool verifies whether zero-load links are found, if yes, then it removes the paths passing through these links from the routing choices and solve the LP system again. Note that the usage of the LP allows Algorithm 4 to solve the problem of joint routing, NC and channel assignment with simulated annealing method.

We assume an ideal MAC layer with both the lossless links and an optimal medium access algorithm, i.e., the channel is fairly assigned to nodes that have a packet for transmission. For simplicity, we assume all channels have the same capacity. Note that the ideal MAC assumption is held for both coding and non-coding schemes, i.e., the performance evaluation for non-coding scheme is done under the assumption of ideal MAC, too. In COPE, as a first testbed deployment of NC, reported that the throughput improvements for a real MAC (802.11 family) is higher than the theoretical value in some situations such as UDP transmissions. The additional gains basically originate from intrinsic unfairness of 802.11 MAC. Using a careful model for an 802.11 like MAC within our theoretical framework is more challenging task and will be the subject of our future work.

6.3 Evaluation results

As mentioned above, we assume the nodes in the network are equipped with a fixed number of interfaces which is denoted by Q in the plots. Further, N is the number of non-overlapping channels in the network.

Evaluation 1: Convergence speed of the proposed algorithm The Fig. 5 demonstrates the convergence speed of the proposed channel assignment algorithm. The configuration is set to be 4 interfaces per node, 4 orthogonal channels and random traffic consisting of 100 demands. The objective is to maximize the throughput of the single path routing by regarding the NC opportunities of both Star and COPE type schemes, namely SP-NC(STAR + COPE). Initially, the algorithm keeps oscillating, attempting to walk out of local optimums. As the temperature is reduced, it gradually approximates a stable solution. The Fig. 5 also compares the convergence of our algorithm with RAC [19]. RAC is an optimization algorithm that jointly optimizes routing, assignment of channels, and coding (henceforth referred to as RAC). It considers only the packet exchange paradigm of COPE-type NC, i.e., the coding is done with the absence of opportunistic listening. Both algorithms start from a similar point of channel assignment (i.e., randomly generated by Algorithm 1) which has an initial throughput equals to 1.63 relative to link capacity. The results show that our algorithm converges to the throughput of 2.41 after 17 iterations. At the same time the solution becomes fixed, the acceptance probability vanishes and remains stable. On the other side, RAC experiences a wide range of oscillations. It reaches to maximum throughput equals to 2.17 at 24th iteration, remains stable at 2.08 for a long time from iteration 37 to 90. Finally, it converges to 2.12 which is not the maximum among 100 iterations. The convergence speed depends on the control parameters and mainly on θ which determines the vanishing speed of acceptance probability. The initial value of $0<\theta<1$ only affects the number of iterations for annealing process, i.e., it becomes high when $\theta \to 1$ and vice versa when $\theta \to 0$. Our empirical setting for these parameters are $\theta = 0.7$, $\tau_0 = 2$, $\tau_F = 10^{-3}$. Note that, for RAC, we set θ to 0.9 (0.87 is recommended in [19]) and used the same values for other parameters. Based on these settings, our algorithm repeats the annealing process at most for 22 iterations while RAC repeats for 70 iterations.

Evaluation 2: Coding for single-path routing and random traffic model Figure 6(a) shows the benefits of coding for single-path routing. Since the routes in single-path are fixed, (i.e., selected on the basis of WCETT metric with $\beta = 0.5$), the throughput improvement basically originates from exploiting the coding opportunities. The improvement for STAR and COPE are approximately equal to 12 and 26 %, respectively relative to non-coding scheme. Further, joint STAR and COPE coding has performance improvement about 34 % relative to single-path routing. This

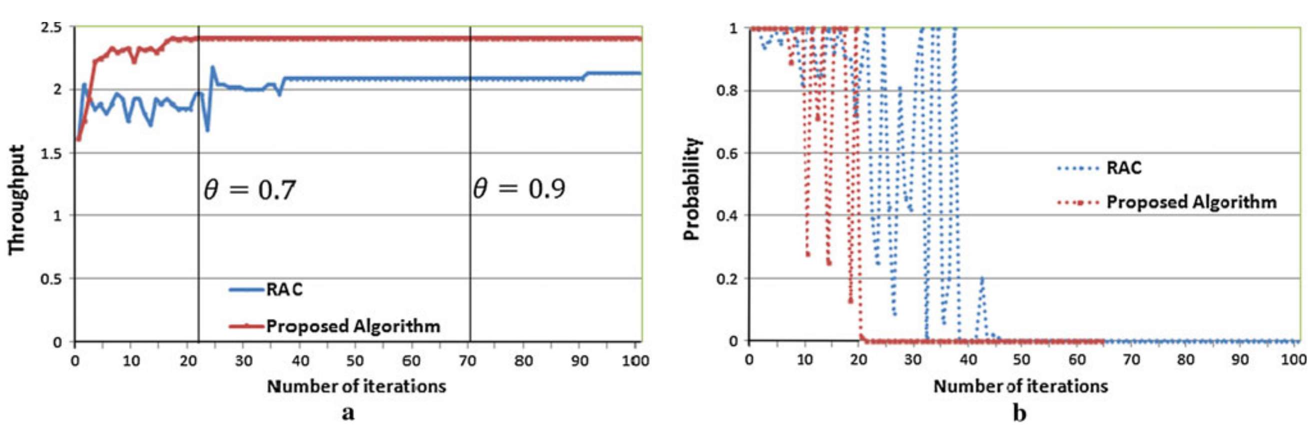

Fig. 5 Convergence speed of the proposed algorithm versus RAC (Q = N = 4). **a**. Convergence of throughput to optimal value. The initial value of θ is set to 0.7/0.9 for RAC/proposed algorithm, **b**. Acceptance probability

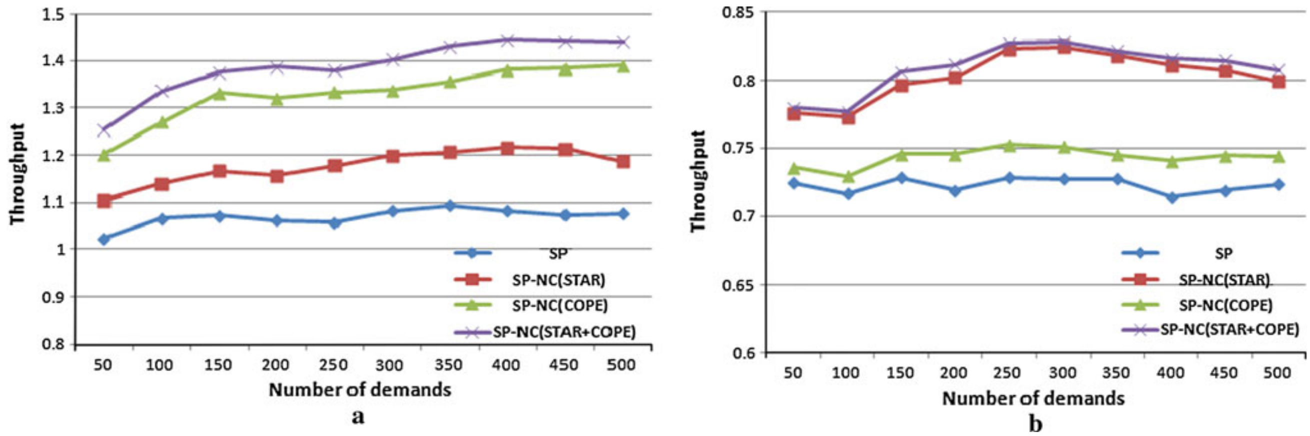

Fig. 6 Throughput normalized to link capacity (Q = N = 2). **a**. Random traffic model, **b**. Directional traffic model

means that Star-NC exploits coding opportunities, different from COPE, which can improve the gain of coding up to 8 % relative to COPE-Type NC scheme.

Evaluation 3: Coding for single-path routing and directional traffic model We repeat the previous evaluation for a directional traffic model. We divide the 7×7 square into two sub squares of size 3.5×7 named as *left* and *right* group. Each flow begins from a node in left group and ends to a node in right group. As a first observation of the results, shown in Fig. 6(b), a significant gap exists between the aggregated throughputs of this model and the results for the random traffic model. We can see that the throughput for this model varies from 0.7 to 0.85, depending on the routing-coding methods, while for the previous model it alters between 1.0 and 1.5.

The results shows that the improvement for COPE is very limited while the gain for Star-NC is significant, i.e., Star-NC outperforms the COPE-type NC. Indeed, the coding opportunities for some COPE-type NC structures such as Alice-Bob are not formed due to the traffic model, i.e., the opportunities for the COPE-type scheme are limited to a series of "X" topologies. Precisely, we can see that COPE's gain is limited to 3 % while both variations of Star-NC have a gain about 12 %. Moreover, we observe that the joint Star and COPE-type coding has no significant benefit over Star-NC alone. Note that this traffic is a typical form of traffic for a wireless mesh networks which provide Internet connectivity for end users. The gateway nodes connect the end users to Internet. Since a wide range of network applications adopt a client/server model in which most of end users act as client, a large portion of traffic is related to the flows from the gateway nodes to the client nodes.

Evaluation 4: Coding for multi-path and optimized single-path routing Multi-path routing selects the paths that minimize the interference among the flows. Since the average node degree in our network topology is relatively high (≈ 6.0), MP always finds multiple paths between each pair of nodes. At the same time, the optimized single path, instead of using only the shortest WCETT metric, tries to find the paths that reduce interference, too. The Fig. 7 depicts the performance of coding for multi-path and optimized single-path routing for both random and directional traffic model. As shown in the Fig. 7(a), MP, itself increases throughput by 35 % relative to SP which is equal to gain for SP-NC(STAR + COPE). By taking the advantage of coding, the joint Star and COPE coding has an approximate gain of 35 % relative to MP. Furthermore, the results show that the performance of the optimized single path is very close to MP and accordingly far from SP, i.e., the difference between the gains of OSP and MP is less than 4 % (relative to SP) for both coding and non-coding variations.

Moreover, Fig. 7(b) shows the results for directional traffic model. We observe that the performance of MP and OSP in non-coding scheme is about 30 % over SP. Moreover, the coding gain for MP and OSP is approximately equal to 12 % which is similar to performance of coding in single-path routing for directional traffic model. In summary, we can argue that the optimized single-path routing has a performance as good as multi-path routing.

Evaluation 5: Influence of the number of channels We study the impact of the number of available channels on the aggregate network throughput. We fix the set of traffic demands to 100 sessions and vary the number of channels from 2 to 12. Figure 8 shows the results respectively for 2 and 3 interfaces per each node. We observe that the throughput has a steep increase till 5 channels and a moderate growth between 5 and 10 channels. From 10 to 12 channels, we do not see a significant improvement with the addition of more channels. It is because of adding excessive channels reduce the number of available links for each node that leads to longer paths and

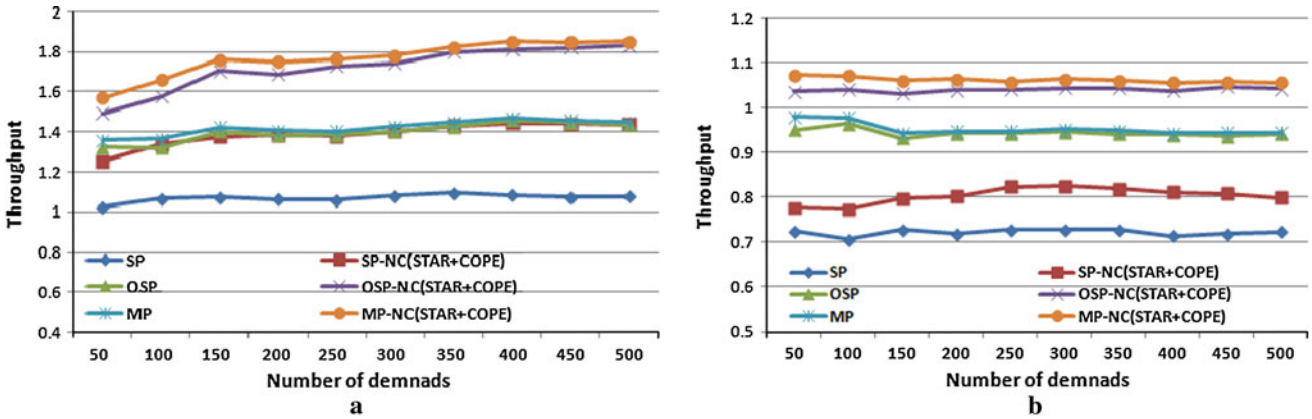

Fig. 7 Performance of coding for different routing strategies (Q = N = 2), **a**. Random traffic model, **b**. Directional traffic model

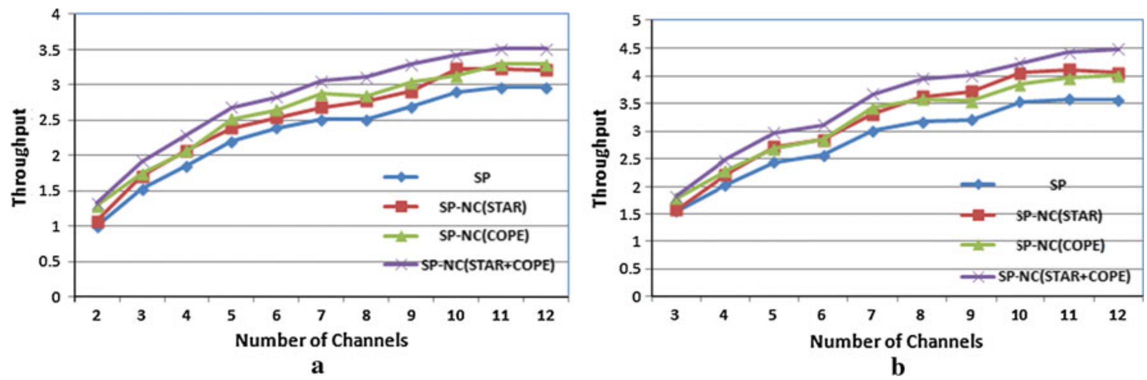

Fig. 8 Aggregate throughput versus number of channels (for a random traffic model consists of 100 demands). **a** 2-interface, **b** 3-interface

correspondingly more interference among the flows. Furthermore, the most of channel assignments with 10 or more channels lead to a disconnected network. To better understanding, suppose a random channel assignment in which every node randomly selects a set of Q channels among N available channels. The probability that two adjacent nodes, namely u and v, share a common channel and thus link (u, v) appears in the network, is equal to:

$$P(Q,N) = P(u\,\text{share a channel with}\,u)$$
$$= \begin{cases} 1 & N < 2Q \\ 1 - \dfrac{\binom{N-Q}{Q}}{\binom{N}{Q}} & N \geq 2Q \end{cases}$$

For example, $P(2, 10) = 0.38$. It means that for $Q = 2$ and $N = 10$, the resulting graph, after channel assignment, either is not connected or a sparse graph which most of the original links are missed due to the absence of a common channel. In other words, most of the links in background graph (i.e., in single-channel) do not appear in induced graph (i.e. in multi-channel system).

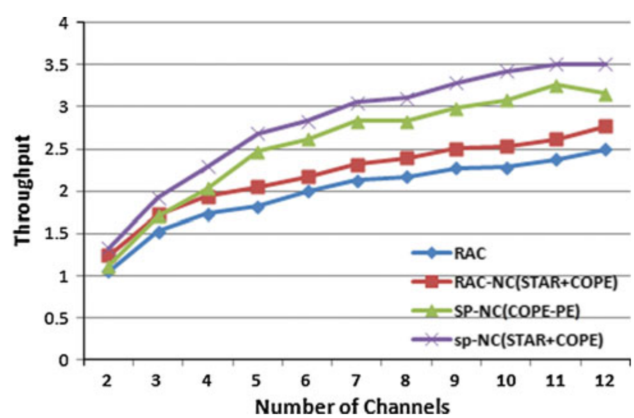

Fig. 9 Comparison of the proposed algorithm versus RAC (random traffic of 100 demands)

Evaluation 6: Comparison with other channel assignment algorithms In Fig. 9(b), we compare the performance of our coding, routing and channel assignment with RAC [19]. We fix the traffic to a random set of 100 demands and vary the number of channels for 2-interface per node setting. The coding of RAC is restricted to COPE packet exchange paradigm. In addition to original RAC, we consider an extension of

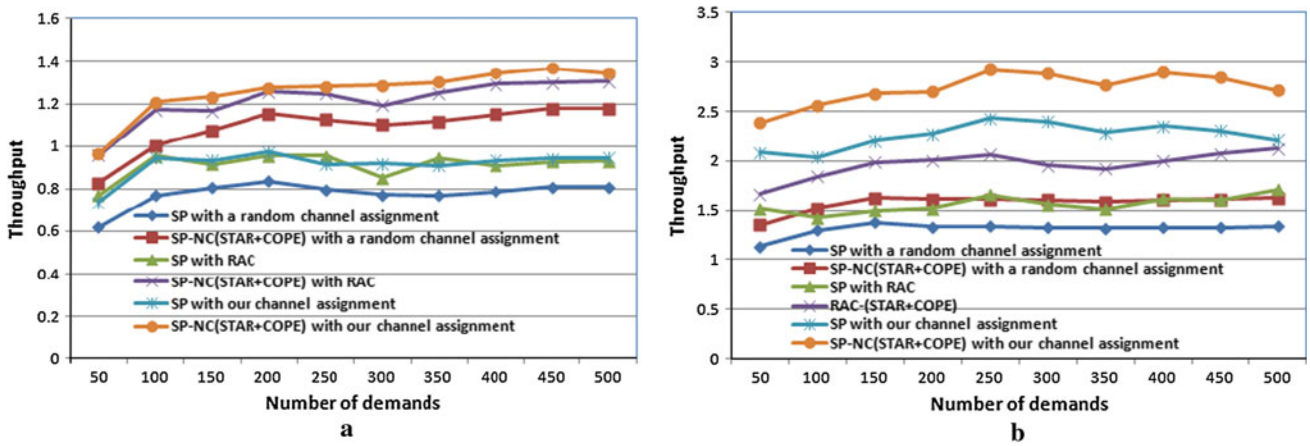

Fig. 10 Comparison of our channel assignment algorithm with both RAC and a randomized algorithm. **a**. 2 interfaces and 5 channels(Q = 2, N = 2), **b**. 2 interfaces and 5 channels (Q = 2, N = 5)

it, referred to as RAC-NC(STAR + COPE), which extends the coding scheme to joint STAR and COPE. We further consider a scenario that limits our algorithm to COPE packet exchange paradigm, referred as SP-NC(COPE-PE). The results show that the aggregate throughput of SP-NC(STAR + COPE) is higher up to 40 % relative to RAC. The extension of RAC, RAC-NC(STAR + COPE), has an average of 10 % performance over RAC. We further observe that our algorithm even with limited coding, i.e. SP-NC(COPE-PE), has an average gain about 12 % relative to extended RAC. Note that the performance gain of our algorithm, relative to RAC, monotonically increases as the number of channels grows. More specifically, there is no significant gap between two algorithms for 2-channels. However, for 12-channels, we can see that our algorithm has an improvement about 40 % over RAC.

We also compare our channel assignment along RAC versus a random channel assignment (Fig. 10). We perform the experiment for two different configurations, 2-channels and 5-channels setting. For both of them, we fix the number of interfaces to 2. Further, we pick a random traffic model for a set of demands varied between 50 and 500. For 2-channels setting, shown in Fig. 10(a), we can see that the performance of our algorithm is approximately equal to RAC for both coding and non-coding schemes, i.e., both them have a performance gain about 18 % over randomized channel assignment. More interestingly, we observe that using of coding opportunities, even in randomized channel assignment, causes a significant improvement (about 40 %) in aggregate throughput. In Fig. 10(b), we repeat the above experiment for 5-channels setting. There is a significant gap (about 40 % in average) between our algorithm with RAC for both coding and non-coding schemes. RAC has an improvement near 25 % over random channel assignment. Indeed, the performance gain of both our algorithm and RAC for 5-channels has growth relative to 2-channels experiment. In summary, we can argue that the efficiency of channel assignment algorithm becomes highlighted as the number of orthogonal channels grows.

7 Discussion and conclusion

In this paper, we analyzed the performance gain of NC in multi-channel/interface wireless networks. For the case of traditional 802.11 mesh networks with multiple interfaces, we derived the potential throughput gain when routing, channel assignment and NC are jointly optimized. Our collaboration scheme can exploit various coding opportunities for both with and without opportunistic listening. Specifically, we considered the Star-NC scheme in addition to COPE-type coding scheme. We provided an LP formulation that naturally combines NC with routing, under arbitrary channel assignment scheme. Using this framework, we proposed an algorithm which assigns channels to the available interfaces with the awareness of both interference and coding opportunities. Evaluation results show that NC can boost the capacity of emerging multi-channel networks.

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