ECE4270
Fundamentals of DSP
Lecture 1
Introduction & Overview

School of Electrical and Computer Engineering

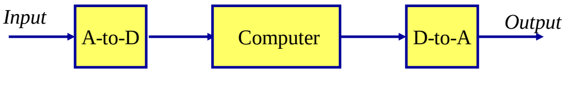
Center for Signal and Image Processing

Georgia Institute of Technology

Overview of Lecture 1

- What is DSP?
- Discrete-time signals
  - Examples
- Discrete-time systems
  - Examples
  - Properties
  - Testing for properties
- Linear time-invariant systems (LTI)

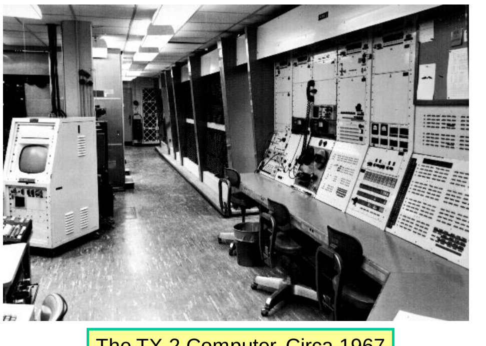
What is DSP??



"That discipline which has allowed us to replace a circuit previously composed of a capacitor and a resistor with two anti-aliasing filters, an A-to-D and a D-to-A converter, and a general purpose computer (or array processor) so long as the signal we are interested in does not vary too quickly."

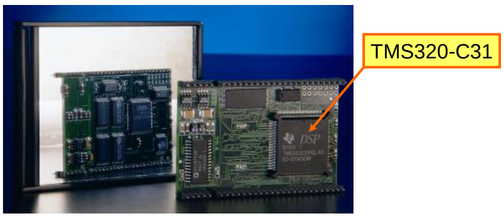
Thomas P. Barnwell, III
Circa 1976

DSP in 1967



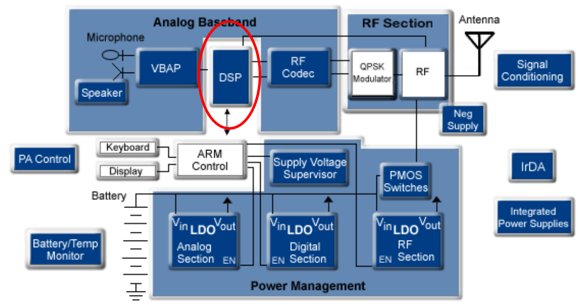
The TX-2 Computer, Circa 1967

DSP Today



MPEG audio encoder/decoder from ASPI

Typical Hot DSP Application



Cellular Phone

Advantages of Digital Representations

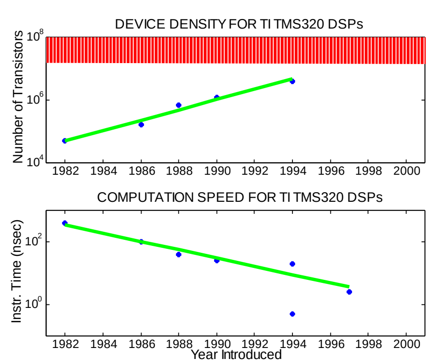
Permanence and robustness of signal representations

Advanced IC technology works well for digital systems

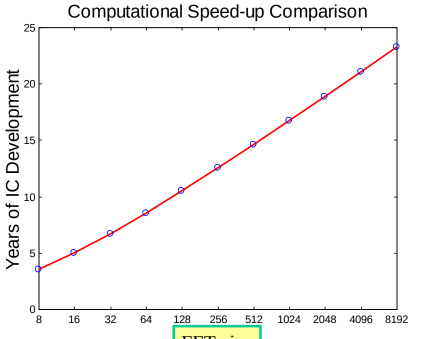
Virtually infinite flexibility with digital systems

* Multi-functionality
* Multi-input/multi-output

Moore's Law for TI DSPs



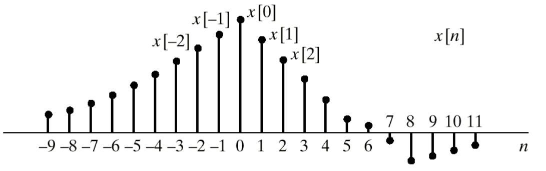
Processor Speed vs Algorithm Cleverness



Number of years of IC development required to achieve FFT speed-up

FFT size

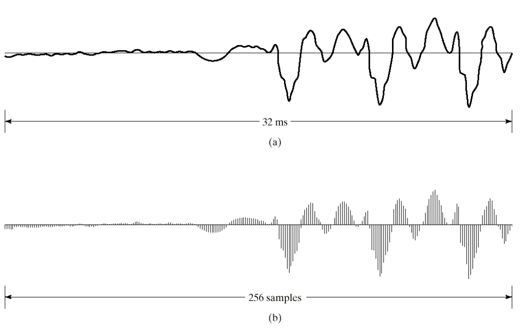
Discrete-Time Signals are Sequences



$x[n]$ denotes the "sequence value at 'time' n"

- Sources of sequences:
  - Sampling a continuous-time signal $x[n]=x_c(nT)$
  - Mathematical formulas

Sampled Speech Waveform

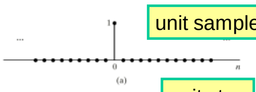


Sampling period is T = 0.125 msec

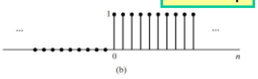
Useful Sequences

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

unit sample



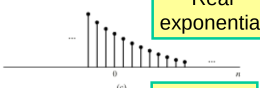
unit step



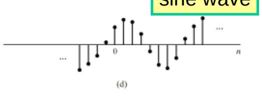
$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

$$x[n] = \alpha^n$$

Real exponential



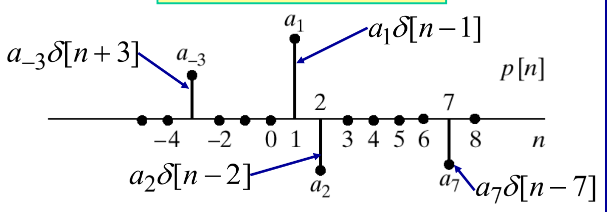
sine wave



$$x[n] = A\cos(\omega_0 n + \phi)$$

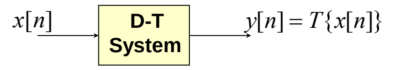
Impulse Representation of Sequences

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$



$$p[n] = a_{-3}\delta[n+3] + a_1\delta[n-1] + a_2\delta[n-2] + a_7\delta[n-7]$$

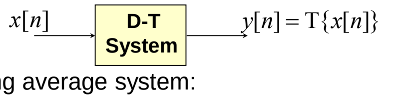
Discrete-Time Systems



- A system transforms an input into an output.
  - Delay: $x[n] \mapsto y[n] = x[n-n_d]$
  - Modulator: $x[n] \mapsto y[n] = x[n]\cos(\omega_0 n)$
  - Squarer: $x[n] \mapsto y[n] = (x[n])^2$
  - Compressor: $y[n] = x[Mn]$ (*downsampler*)
  - Expander: (*upsampler*)

$$x[n] \mapsto y[n] = \begin{cases} x[n/L], & n = 0, \pm L, \ldots \\ 0, & \text{otherwise} \end{cases}$$

More Discrete-Time Systems



L-point moving average system:

$$y[n] = \frac{1}{L}\sum_{k=0}^{L-1} x[n-k] = \frac{1}{L}(x[n] + x[n-1] + \cdots + x[n-L+1])$$

Accumulator system:

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

Properties of D-T Systems

- A system is linear if and only if

$$\mathrm{T}\{ax_1[n] + bx_2[n]\} = a\mathrm{T}\{x_1[n]\} + b\mathrm{T}\{x_2[n]\}$$

- A system is time-invariant if and only if

$$x_1[n] = x[n-n_d] \Rightarrow y_1[n] = y[n-n_d]$$

- A system is causal if and only if

$$y[n] \text{ depends only on } x[k] \text{ for } k \le n$$

- A system is BIBO stable if every bounded input produces a bounded output; i.e.,

$$\text{when } |x[n]| < B_x < \infty \text{ for all } n, \quad \text{then } |y[n]| < B_y < \infty \text{ for all } n$$

Moving Averager: $y[n] = (1/L)\sum_{k=0}^{L-1} x[n-k]$

Linearity: yes

$$\frac{1}{L}\sum_{k=0}^{L-1}(ax_1[n-k]+bx_2[n-k]) = a\left(\frac{1}{L}\sum_{k=0}^{L-1}x_1[n-k]\right)+b\left(\frac{1}{L}\sum_{k=0}^{L-1}x_2[n-k]\right)$$

Time-invariance: yes

$$\frac{1}{L}\sum_{k=0}^{L-1}x[n-k-n_d] = \frac{1}{L}\sum_{k=0}^{L-1}x[(n-n_d)-k] = y[n-n_d]$$

Causality: yes

$$y[n] = \frac{1}{L}(x[n]+x[n-1]+\cdots+x[n-L+1])$$

Stability: yes

$$|y[n]| = \left|\frac{1}{L}\sum_{k=0}^{L-1}x[n-k]\right| \le \frac{1}{L}\sum_{k=0}^{L-1}|x[n-k]| \le B_x$$

Down Sampler: $y[n] = x[Mn]$

Linearity yes

$$ax_1[Mn]+bx_2[Mn] = ay_1[n]+by_2[n]$$

Time-invariance no

$$y_1[n] = x_1[Mn] = x[Mn-n_d] \ne y[n-n_d] = x[M(n-n_d)]$$

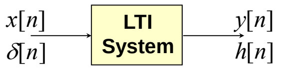
Causality no

$$y[-1] = x[-M], \text{ but } y[+1] = y[M]$$

Stability yes

$$|y[n]| = |x[Mn]| \le B_x$$

LTI Discrete-Time Systems



- Linearity (superposition):

$$\mathrm{T}\{ax_1[n]+bx_2[n]\} = a\mathrm{T}\{x_1[n]\}+b\mathrm{T}\{x_2[n]\}$$

- Time-Invariance (shift-invariance):

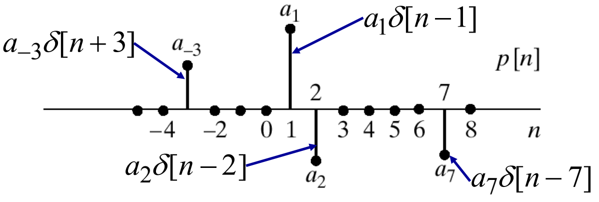
$$x_1[n] = x[n-n_d] \quad \Rightarrow \quad y_1[n] = y[n-n_d]$$

- LTI implies discrete convolution:

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = x[n]*h[n] = h[n]*x[n]$$

Impulse Representation of Sequences

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$



$$p[n] = a_{-3}\delta[n+3]+a_1\delta[n-1]$$
$$+a_2\delta[n-2]+a_7\delta[n-7]$$

LTI Discrete-Time Systems

$x[n]$ → **LTI System** → $y[n]$

"impulse" $\delta[n]$ → $h[n]$ impulse response

$x[k]\delta[n-k]$ → $x[k]h[n-k]$

$\sum_{k=-\infty}^{\infty} x[k]\delta[n-k] = x[n]$ → $\sum_{k=-\infty}^{\infty} x[k]h[n-k] = y[n]$

convolution sum

$$y[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m] = x[n]*h[n] = h[n]*x[n]$$

Discrete Convolution

- Two ways to look at it:
  - As the representation of the output as a sum of delayed and scaled impulse responses.

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = x[0]h[n] + x[1]h[n-1] + \ldots$$
$$+ x[-1]h[n+1] + \ldots$$

  - As a computational formula for computing *y*[*n*] ("*y* at time *n*") from the entire sequences *x* and *h*.
    - Form $x[k]h[n-k]$ for $-\infty < k < \infty$ for fixed n.
    - Sum over all k to produce $y[n]$.
    - Repeat for all n.

"Flipping and Shifting"