ECE4270
Fundamentals of DSP

Lecture 23

IIR and FIR Filter Design

School of ECE
Center for Signal and Information Processing
Georgia Institute of Technology

Overview of Lecture

- Butterworth, Chebyshev and Elliptic IIR filters
- Design of FIR filters by the window method.
  - Windowing in time- and frequency-domains
  - Linear phase FIR design using windows
  - The Kaiser window formulas
- Design of differentiator
- Multiband design
- Introduction to Optimum Filter Design

A Design Example (I)

- The D-T specifications are:

$$0.99 \le |H(e^{j\omega})| \le 1.01, \qquad |\omega| \le 0.4\pi$$

$$|H(e^{j\omega})| \le 0.001, \quad 0.6\pi \le |\omega| \le \pi$$

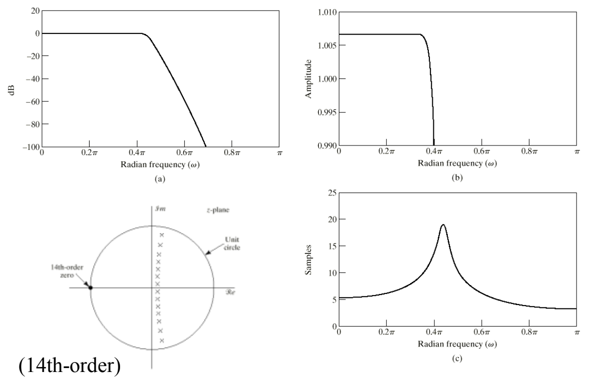
i.e., $\omega_p = \Omega_p T = 0.4\pi$ and $\omega_s = \Omega_s T = 0.6\pi.$

- The continuous-time prototype filter $H_c(j\Omega)$ must satisfy:

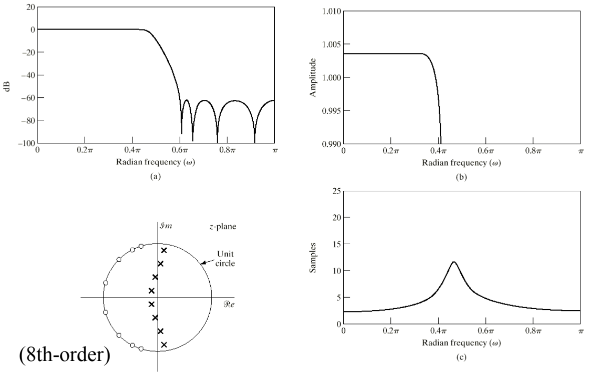
$$0.99 \le |H_c(j\Omega)| \le 1.01, \qquad |\Omega| \le \frac{2}{T_d}\tan(0.4\pi/2)$$

$$|H_c(j\Omega)| \le 0.001, \quad \frac{2}{T_d}\tan(0.6\pi/2) \le |\Omega| < \infty$$

Butterworth Approximation (III)



Chebyshev Approximation (IV)



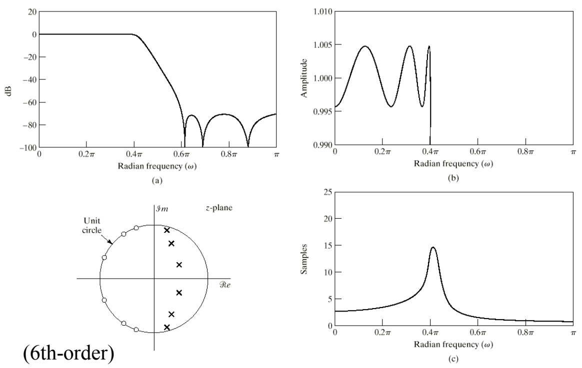
(8th-order)

Elliptic Approximation in MATLAB (V)

```
»[N,wp]=ellipord(.4,.6,-20*log10(.99),-20*log10(.001))
N =
    6
wp =
   0.4000

»[b,a]=ellip(N,-20*log10(.99),-20*log10(.001),wp)
b =
   0.0208    0.0590    0.1068    0.1265    0.1068    0.0590    0.0208
a =
   1.0000   -1.9585    2.8916   -2.5155    1.5627   -0.5906    0.1148
```


Elliptic Approximation (VI)

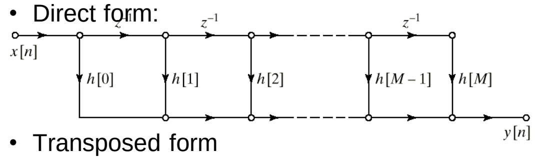


(6th-order)

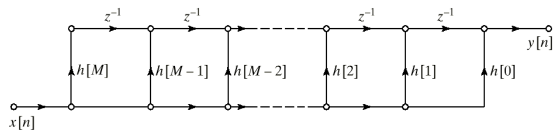
FIR Filter Structures

$$y[n] = \sum_{k=0}^{M} h[k]x[n-k] \qquad H(z) = \sum_{k=0}^{M} h[k]z^{-n}$$

- Direct form:

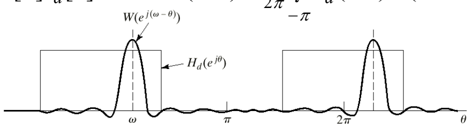


- Transposed form

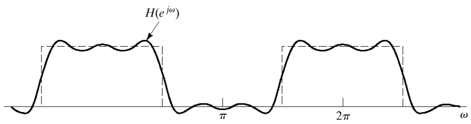


FIR Filter Design: The Window Method

$$h[n]=w[n]h_d[n] \quad \Leftrightarrow \quad H(e^{j\omega})=\frac{1}{2\pi}\int_{-\pi}^{\pi}H_d(e^{j\theta})W(e^{j(\omega-\theta)})d\theta$$



(a)



(b)

FIR : Linear Phase in Window Design - I

- Choose a symmetric causal window such that

$$w[M-n]=w[n] \quad \Leftrightarrow \quad W(e^{j\omega})=W_e(e^{j\omega})e^{-j\omega M/2}$$

$$w[n]=0, \quad \text{for } n<0 \text{ and } n>M \text{ and}$$

and either a symmetric ideal impulse response

$$h_d[M-n]=h_d[n] \quad \Leftrightarrow \quad H_d(e^{j\omega})=H_e(e^{j\omega})e^{-j\omega M/2}$$

or an anti-symmetric impulse response

$$h_d[M-n]=-h_d[n] \quad \Leftrightarrow \quad H_d(e^{j\omega})=jH_o(e^{j\omega})e^{-j\omega M/2}$$

- Then it follows that

$$h[n]=w[n]h_d[n]=\pm h[M-n], \quad 0\le n\le M.$$

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FIR: Linear Phase in Window Design - II

- $h[n]=w[n]h_d[n] \quad \Rightarrow$

$$H(e^{j\omega})=\frac{1}{2\pi}\int_{-\pi}^{\pi}H_d(e^{j\theta})W(e^{j(\omega-\theta)})d\omega$$

$$H(e^{j\omega})=\frac{1}{2\pi}\int_{-\pi}^{\pi}H_e(e^{j\theta})e^{-j\theta M/2}W_e(e^{j(\omega-\theta)})e^{-j(\omega-\theta)M/2}d\theta$$

$$=\left(\frac{1}{2\pi}\int_{-\pi}^{\pi}H_e(e^{j\theta})W_e(e^{j(\omega-\theta)})d\theta\right)e^{-j\omega M/2}$$

$$H(e^{j\omega})=A_e(e^{j\omega})e^{-j\omega M/2} \quad \text{where,}$$

$$A_e(e^{j\omega})=\frac{1}{2\pi}\int_{-\pi}^{\pi}H_e(e^{j\theta})W_e(e^{j(\omega-\theta)})d\omega$$

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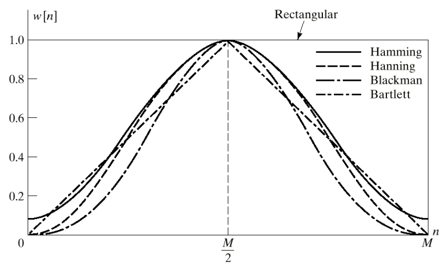
Some Common Windows



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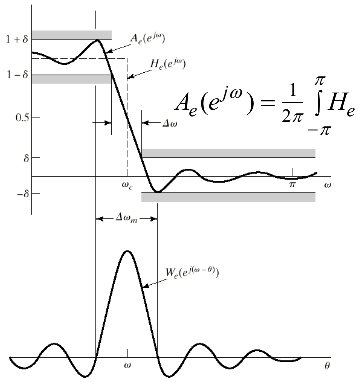
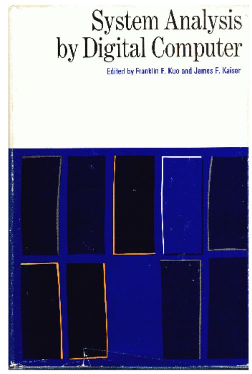
Details of the Window Approximation



$$A_e(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_e(e^{j\theta})W_e(e^{j(\omega-\theta)})d\theta$$

The phase is exactly linear with delay *M* /2

Jim Kaiser



Kaiser Window Design Method

$$w[n] = \begin{cases} \dfrac{I_0[\beta(1-[(n-\alpha)/\alpha]^2)^{1/2}]}{I_0(\beta)}, & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$$

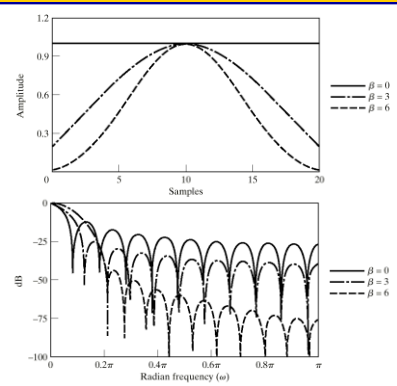
$$\alpha = M/2$$

$$\Delta\omega = \omega_s - \omega_p \quad \text{and} \quad A = -20\log_{10}\delta$$

$$M = \frac{A-8}{2.285\Delta\omega} \quad \Rightarrow \quad \text{required to meet specs}$$

$$\beta = \begin{cases} 0.1102(A-8.7), & A > 50 \\ 0.5842(A-21)^{0.4} + 0.07886(A-21), & 21 < A < 50 \\ 0.0 & A < 21 \end{cases}$$

Kaiser Windows



Lowpass Filter Design Example

- Ideal filter:

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega M/2} & |\omega| < \omega_c, \\ 0 & \omega_c < |\omega| \le \pi \end{cases}$$

$$h_d[n] = \frac{\sin \omega_c (n - M/2)}{\pi (n - M/2)}$$

- Specifications:

$$\omega_p = 0.4\pi,\ \omega_s = 0.6\pi,\ \delta_1 = 0.01,\ \delta_2 = 0.001$$

$$\omega_c = \frac{\omega_p + \omega_s}{2} \quad \text{since transition is symmetric}$$

$$A = 20\log_{10}(.001) = 60 \quad \text{since error is symmetric}$$

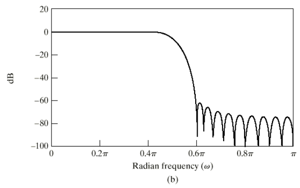
Kaiser Window Design Example

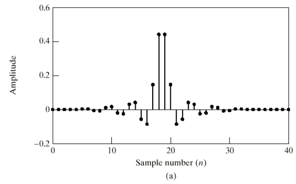
$$\omega_p = 0.4\pi,\ \omega_s = 0.6\pi$$

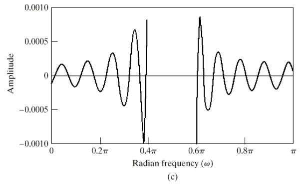
$$\Delta\omega = 0.2\pi,$$

$$A = -20\log_{10}(.001) = 60$$

$$M = \left\lceil \frac{52}{2.285(.2\pi)} \right\rceil = 37$$







Digital "Differentiator"

- Suppose that we want to design an FIR filter such that

$$H_{\text{eff}}(j\Omega) = j\Omega, \quad |\Omega| < \frac{\pi}{T}$$

- The required digital filter must approximate

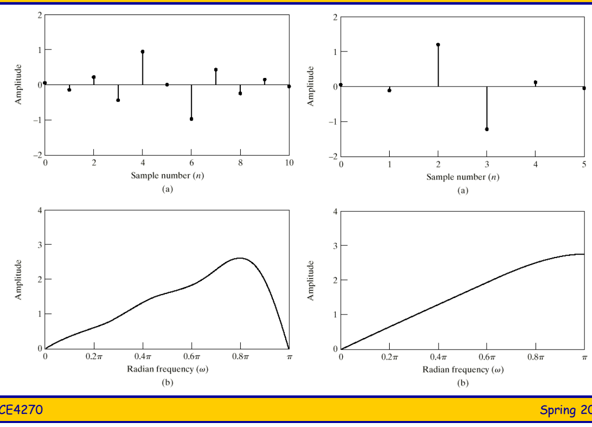
$$H(e^{j\omega}) = \frac{j\omega}{T} e^{-j\omega M/2}, \quad |\omega| < \pi$$

- The desired impulse response is

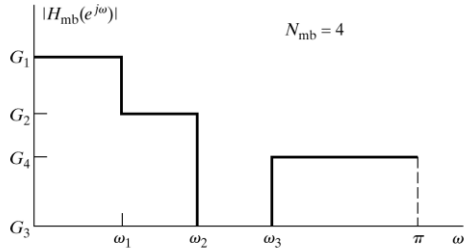
$$h_d[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left(\frac{j\omega}{T}\right) e^{-j\omega M/2} e^{j\omega n} d\omega$$

$$h_d[n] = \frac{\cos[\pi(n - M/2)]}{(n - M/2)T} - \frac{\sin[\pi(n - M/2)]}{\pi(n - M/2)T}, \quad -\infty < n < \infty$$

Kaiser Window Differentiators

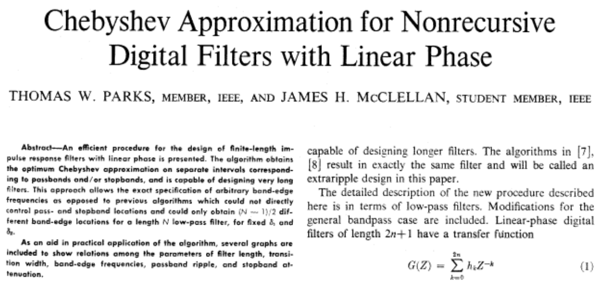


General Frequency Selective Filter



$$h_{mb}[n] = \sum_{k=1}^{N_{mb}} (G_k - G_{k+1}) \frac{\sin \omega_k (n - M/2)}{\pi (n - M/2)}$$

Parks and McClellan, 1972



Chebyshev Approximation for Nonrecursive Digital Filters with Linear Phase

THOMAS W. PARKS, MEMBER, IEEE, AND JAMES H. McCLELLAN, STUDENT MEMBER, IEEE

Abstract—An efficient procedure for the design of finite-length impulse response filters with linear phase is presented. The algorithm obtains the optimum Chebyshev approximation on separate intervals corresponding to passbands and/or stopbands, and is capable of designing very long filters. This approach allows the exact specification of arbitrary band-edge frequencies as opposed to previous algorithms which could not directly control pass- and stopband locations and could only obtain $(N-1)/2$ different band-edge locations for a length N low-pass filter, for fixed δ_1 and δ_2.

As an aid in practical application of the algorithm, several graphs are included to show relations among the parameters of filter length, transition width, band-edge frequencies, passband ripple, and stopband attenuation.

capable of designing longer filters. The algorithms in [7], [8] result in exactly the same filter and will be called an extraripple design in this paper.

The detailed description of the new procedure described here is in terms of low-pass filters. Modifications for the general bandpass case are included. Linear-phase digital filters of length $2n+1$ have a transfer function

$$G(Z) = \sum_{k=0}^{2n} h_k Z^{-k} \quad (1)$$

T. W. Parks and J. H. McClellan, *IEEE Trans. Circuit Theory*, CT-19, pp. 189-194, March, 1972.

Optimum FIR Filter