GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

ECE 6605
Information Theory and Inference

Assigned: Thursday, Aug. 23, 2018
Due: Thursday, Sept. 6, 2018
Due Section Q (Video Student): Thursday, Sept. 13, 2018

Problem Set #1

Problem 1-5: Solve the questions 2.1 (from page 43), 2.4, 2.7 (part *a* only), 2.17 and 2.18 from Chapter 2 of the textbook (the second edition).

Problem 6: Consider the discrete memoryless source (A, B, C, D) with probabilities $(1/2, 1/4, 1/8, 1/8)$, with the binary source code:

$$\begin{array}{ccc} A & \rightarrow & 00 \\ B & \rightarrow & 01 \\ C & \rightarrow & 10 \\ D & \rightarrow & 11 \end{array}$$

(a) With this code, calculate the probability of the channel seeing a "0" and a "1".

(b) See if you can rearrange the four codewords $\{00, 01, 10, 11\}$ differently, so that these probabilities are both 1/2.

(c) Explain why the rearranged code you found in part (b) is still not much good, despite Shannon's prescription, which says that the code should cause the channel to see a sequence of independent, equally likely 0's and 1's.

Problem 7: Let X, Y and Z be three discrete random variables. Using Jensen's inequality, or otherwise (e.g., the chain rule), show that $I(X, Y; Z) \geq I(Y; Z)$. When does the equality hold?

Problem 8: Let X_1, X_2 be discrete random variables drawn according to probability mass functions $p_1(\cdot)$ and $p_2(\cdot)$ over the respective alphabets $\mathcal{X}_1 = \{1, 2, \ldots, m\}$ and $\mathcal{X}_2 = \{m + 1, 2, \ldots, n\}$. Let

$$X = \begin{cases} X_1 & \text{with probability } \alpha \\ X_2 & \text{with probability } 1 - \alpha \end{cases}$$

Find $H(X)$ in terms of $H(X_1)$ and $H(X_2)$ and α.

Hint: define a function of X as :

$$\gamma = f(X) = \begin{cases} 1 & \text{when } X = X_1 \\ 2 & \text{when } X = X_2 \end{cases}$$

and use $H(X, \gamma)$.

Problem 9: Let assume the time is discretized to time slots or epochs. At each time epoch we throw a biased coin (with probability 1/4 as being Tail). Let X be the waiting time (i.e., the number of time epochs) for the first heads to appear in successive flips of a fair coin. Thus, the distribution of the waiting time for the first head is given by $Pr\{X = i\} = (3/4)(1/4)^{(i-1)}$.Let S_n be the waiting time for the n^{th} head to appear. Thus, we have:

$$S_0 = 0$$

$$S_{n+1} = S_n + X_{n+1} \quad \text{for } n = 0, 1, 2, \ldots,$$

where X_i $X_1, X_2, X_3, \ldots$ are iid.

(a) Determine $H(X)$.

(b) Determine $H(S_1, S_2, \ldots, S_n)$.