GEORGIA INSTITUTE OF TECHNOLOGY
School of Electrical and Computer Engineering

ECE 6605
Information Theory

Assigned: Thursday, Sep. 6, 2018
Due: Thursday, Sept. 20, 2018
Due Section Q (Video Student): Thursday, Sept. 27, 2016

Problem Set #2

Problem 1-4: Solve questions 3.1, 3.7, 3.8, and 3.13 from Chapter 3 of the textbook (the second edition). For the definition of the smallest set in Problem 3.13 refer to page 62 of the textbook)

Problem 5: Suppose that a six sided die is thrown n times; assume that n is large. The faces of the die are numbered 1 through 6, and each is equally likely. Find a good upper bound on the probability that the average of the numbers computed over the n throws is less than 2.5. Hint: use Chebyshev's inequality.

Problem 6-7: Solve questions 3.9, and 3.10 from Chapter 3 of the textbook (the second edition).

Problem 8: Suppose that X is a random variable with a probability distribution (i.e., probability that $X = k$) given by:

$$P_X(k) = Prob(X = k) = (1 - \lambda)\lambda^k \quad for \;\; k \geq 0$$

where $0 < \lambda < 1$ and k is a non-negative integer (and hence X can take any non-negative integer value). To answer this question, note that the AEP theorem we proved for a finite-alphabet random variable also holds with the same formulation for a random variable with infinite but countable alphabet.

(a) Compute $H(X)$.

(b) Describe the typical set for i.i.d sequences of length n: $X_1, X_2, \ldots, X_n$. (Characterize your description of the set clearly by specifying which length n sequences will be in the typical set $A_\epsilon^{(n)}$).

Problem 9: Let $X_1, X_2, \ldots, X_n$ be n random variables taking values in the two-element set $\{0, 1\}$, determined as follows. First, flip a fair coin: If the outcome is heads, set $X_1 = 0$; if it is tail, set $X_1 = 1$. Now roll a fair die; if the outcome is a number from

$\{1, 2, 3, 4\}$, set $X_2 = X_1$; if the outcome is 5 or 6, set $X_2 = \overline{X_1}$ (here $\overline{X_1}$ denotes the complement of X_1). Similarly, roll the die again to determine X_3 from X_2 (changing if the die shows 5 or 6), once again to determine X_4 from X_3,..., and one last time to determine X_n from X_{n-1}.

(a) Compute $H(X_1)$.

(b) Compute $H(X_n)$.

(c) Compute $H(X_1, X_2, \ldots, X_n)$.

(d) Compute $I(X_1; X_2)$.

(e) **(6 points)** Define n random variables $Y_1, Y_2, \ldots, Y_n$ as:

$$Y_1 = X_1 \quad \text{and} \quad Y_i = X_i \bigoplus Y_{i-1} \;\text{ for }\; i = 2, \ldots, n$$

Here $\bigoplus$ is the XOR (logic) operations.

Compute $H(Y_1, Y_2, \ldots, Y_n)$ and $H(Y_1, Y_2, \ldots, Y_n | X_1, X_2, \ldots, X_n)$.