FTS: A Distributed Energy-Efficient Broadcasting Scheme Using Fountain Codes for Multihop Wireless Networks

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***Abstract*—We investigate the problem of reliable and energy-efficient one-to-all broadcasting in multihop wireless networks, and propose fractional transmission scheme (FTS) – a low-complexity and scalable broadcasting scheme. FTS exploits the broadcasting nature of wireless channels and random encoding of rateless codes to reduce energy consumption while ensuring reliable delivery of packets to all nodes in the network. In the proposed scheme, different neighbors of a node share the responsibility of transmitting packets by sending only a fraction of encoded packets required by the node to successfully receive the data sent by the source. A detailed analysis of the performance of FTS is presented for grid and random deployment networks. Further, extensive simulations compare our scheme with present energy-efficient methods such as random linear coding, multipoint relaying, dominant pruning, and broadcast incremental power scheme. Simulations reveal that FTS offers good performance and adaptability at a low computational cost.**

***Index Terms*—Broadcasting, energy efficiency, multihop wireless networks, rateless codes, network coding.**

I. Introduction

EFFICIENT network-wide broadcasting is an important issue in wireless networks. Important performance metrics of a broadcasting scheme include reliability, energy efficiency, complexity, scalability, and latency. Based on the specific application, some metrics are more important than others. For example, when updating software in nodes of a network, reliability is important, while latency may have less importance. However, for real-time sensing, latency and reliability are of paramount importance, whereas in battery-powered sensor networks, energy efficiency is an important metric.

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A large volume of work on efficient communication strategies over multihop networks is available, and a good survey of the protocols and tradeoffs can be found in [1] and [2]. However, our focus is on the setting where a large number of packets have to be *reliably* and (*energy-*)*efficiently* broadcast in a multihop wireless network. Here, we use *reliability* to indicate that all nodes in the network must be able to retrieve the data completely. Broadcasting has been explored using two broad approaches depending on the processing capabilities of nodes in the network.

In the first approach, where nodes are assumed to act only as relays, reliable and energy-efficient broadcasting is equivalent to the problem of finding a minimum-connected dominating-set (MCDS) for the corresponding network graph. However, determining an MCDS is an NP-complete problem [3] even if a centralized algorithm utilizing the full knowledge of the graph topology is employed. Some heuristic algorithms for tackling this problem have been proposed in [4]–[8].

In the second model, each node has the capacity to relay and to perform local processing and coding. This model was first exploited in [9] and their work opened a new research area known as network coding. Considerable work has been done in this area including [10], [11] and references therein. In this model, the problem is solvable by a polynomial-time algorithm assuming that the network is directed. However, this assumption is restrictive, since wireless networks consisting of nodes with omnidirectional antennas are naturally undirected. The issue of finding optimal directions for the edges is a difficult problem, since the number of direction assignments is exponential in the number of edges in the network.

Network coding (NC)[†] approach to efficient broadcast has been studied extensively when the cost criterion is a separable function of the flow on various directed edges of the network. In this case, the problem decouples into two subproblems [10]. The first subproblem is to determine the subgraph over which the actual network coding has to be performed and the packet flow rate on each link in the subgraph. The second subproblem is to determine the code to be used over the selected subgraph.

Optimal subgraph selection and edge flow identification for a separable cost function is achieved by an elaborate linear programming routine. Constraints are devised for each triple (v, J, t), where v runs through all nodes, J runs through all

[†]Although network coding refers to any operation from simple relaying to complicated intermediate processing, we use this term to refer to network coding as defined in [11].

subsets of outgoing edges of v, and t runs through all sinks in the communication strategy. Thus, the number of constraints for optimization for broadcast over a directed network of n nodes with an average out-degree J is about $n^2 2^J$ [11], which is impractical even for moderately dense networks.

To implement the packet flow rate identified from the optimization routine, *random linear coding* (RLC) is employed [12]. In RLC, nodes generate random linear combinations of received packets by treating packets as symbols over a vector space of a finite field of size q [10]. To be efficient, a large q must be chosen. A drawback of this coding scheme is that the decoding at each node involves Gaussian elimination, which has a computational complexity that is polynomial in the number of packets. However, instead of using RLC, practical coding techniques can also be employed at each node. *Rateless erasure coding* is an attractive option that offers low-complexity encoding and decoding [13]–[15]. The encoding is a low-weight packet-level addition of input packets over the binary field, and the decoding is performed by a simple iterative decoder for erasure channels. In [15], [16], schemes based on rateless codes for reliable multicast and broadcast in lossy *single-hop* networks is studied. In such networks, coding is optimal for all clients independent of their packet loss rates, and no prior knowledge about the channel status is needed. In multihop wireless networks, though optimality depends on both the routing pattern and the coding scheme employed, straightforward application of rateless codes to simple multihop situations like the relay channel illustrate the robustness these codes offer [17], [18].

A good broadcasting scheme for wireless networks must ensure that (a) packets are conveyed efficiently through the network, and (b) redundant transmissions are minimized. In RLC, these issues are addressed by optimizing edge flows and by generating random linear combinations over a large field. Note that issue (b) can also be addressed by employing rateless coding with a *decode, re-encode and forward* approach. Once the original data is retrieved, nodes can perform rateless encoding of the original data and transmit newly encoded packets. However, to address issue (a), rateless coding must be appended with a means of practical subgraph selection. Schemes such as FBcast [19], CRBcast [20], and that of [21] utilize practical routing schemes such as probabilistic routing in conjunction with source encoding and/or intermediate node processing to achieve this goal. While some of these are network-unaware approaches, our approach combines: (1) a novel local load-sharing approach to flow selection exploiting hop-distance information, and (2) rateless coding using the decode, re-encode and forward approach. The following section summarizes our contribution.

A. Contribution of The Paper

We propose *fractional transmission scheme* (FTS) – a low-complexity scheme for reliable and energy-efficient one-to-all broadcasting in stationary multihop wireless networks that exploits the availability of neighborhood information around each node. Each node is assumed to know its hop-distance from the source, and that of its neighbors. In FTS, by using hop-distance information, each node ascertains the fraction of data it has to send to a select subset of neighbors. Further, to reduce unwanted redundant transmissions, nodes operate in a *decode, re-encode and forward* mode. Additionally, if each node knows its geographical location and that of its neighbors, FTS can be improved by allowing each node to adjust its transmission radius depending on the farthest node to which it needs to forward packets.

Analysis of FTS in grid and random deployment networks is presented. In grid networks, the analytical results are compared with the corresponding optimal case. For random deployment networks, bounds on the asymptotic order of cost of transmission offered by FTS are established. Though the upper bound is based on a conjecture motivated by simulations, the bound was found to be tight for large graphs. Next, extensive simulation results are provided for grid and random deployment networks. These results suggest that FTS offers reliability, scalability and ease of implementation with a performance that is comparable with competitive techniques such as NC, Broadcast Incremental Power (BIP) [6], Multipoint Relaying (MPR) [7], and Dominant Pruning (DP) [8].

II. Brief Review of Rateless Coding

Rateless codes are a new class of error-control codes. LT codes [14], raptor codes [15], and online codes [13] are examples of such codes. The idea behind rateless codes is that each receiver collects encoded data packets until it can decode successfully. Unlike traditional codes, rateless codes on erasure channels need no channel information, and are therefore suitable candidates for applications where the channel erasure probability is unknown. Another attractive feature of rateless codes is their low-complexity encoding and decoding algorithms.

To transmit n_p packets, the encoder generates encoded packets by performing random packet-level low-weight linear combinations of input packets over the binary field. To generate an encoded packet, the encoder first selects a weight W using an optimized distribution $\Omega(x) = \sum_{i\in\mathbb{N}} \Omega_i x^i$, where $\Omega_i = \Pr[W = w]$ [15]. It then selects W data packets, which are XORed componentwise to generate an encoded data packet. This process is repeated until all decoders receive sufficient number of encoded packets to decode the input data packets. In general, the number of output packets required for guaranteeing a high probability of successful decoding of n_p input packets is expressible as γn_p, where $\gamma \geq 1$ is the rateless overhead. The decoding process is similar to that of other sparse graph codes such as LDPC codes [22]. To construct the sparse graph over which message-passing must be performed, information about the input data packets used to generate each encoded packet must be provided to the decoder. One way to ensure this is by appending the required packet information to the header of encoded packets using an average of $\Omega'(1)\lceil\log_2 n_p\rceil$ bits. Note that this packet overhead is much smaller than the corresponding overhead of RLC, which is $n_p\lceil\log_2 q\rceil$ bits.

III. Network Models and Terminologies

For analytical tractability, we consider the following setup. A network of n static nodes with omnidirectional antennas and a transmission range r is assumed. Such a wireless network can be modeled by a geometric undirected graph $G(V, E)$, where V, E are the set of nodes and edges, respectively. Two nodes u and v share an edge if and only if the Euclidean distance $d(u, v)$ between u and v is less than or equal to r. We also assume that r is large enough so that $G(V, E)$ is connected. The neighborhood $N_r(u)$ of a node u is the set of all nodes that are within a distance of r units away from u i.e., $N_r(u) := \{v \in V : \mathbb{I}_r(d(u, v)) = 1\}$, where $\mathbb{I}_r(x) = 1$ if $0 < x \le r$, and $\mathbb{I}_r(x) = 0$ otherwise. Also, for each node $\partial_u := |N_r(u)|$. Lastly, $B(u, r)$ denotes the closed Euclidean ball of radius r around the node u.

The wireless nature of the network is modeled by the assumption that any message that a node u transmits is heard by all nodes in $N_r(u)$. Also, we assume that the network contains a unique source node s and $\mathbf{H} : V \longrightarrow \mathbb{N} \cup \{0\}$ denotes the *hop-distance* function that maps each node v to the hop-length of the shortest-hop path connecting s to v. Finally, we assume that the network is equipped with a one-to-one node identifier function $id : V \longrightarrow \mathbb{N}$ and each node v knows $id(v)$. For simplicity, we assume that id satisfies for any $u, v \in V$ and $u \in N_r(v)$, if u is closer to the source than v then $id(u) < id(v)$.

As in [6], we only consider the energy expended on RF transmissions as the cost criterion for energy efficiency. The cost for sending a packet by a node with transmission range r is taken to be r^2. We also denote the *number of transmissions per packet per node* by $\mathcal{N}_{/p/n}$ and the corresponding *energy consumption per packet per node* by $\mathcal{C}_{/p/n}$. Note that if m_v packets are losslessly sent by each node v with a transmission range r_v, then $\mathcal{N}_{/p/n} := \frac{1}{n}\sum_v m_v$, where as $\mathcal{C}_{/p/n} := \frac{1}{n}\sum_v m_v r_v^2$.

In simulations, transmissions in networks are assumed to be subject to distance attenuation and Rayleigh fading. Therefore, when a node u with a nominal transmission range r transmits, the signal-to-noise (SNR) of the signal received at a node v with distance $d(u, v)$ from the node u is $\lambda r^2/d(u, v)^\alpha$, where λ is an exponentially-distributed random variable with unit mean, and α is an attenuation parameter called *path loss*. The value of α is usually between 2 and 4 depending on the characteristics of the channel. In this work we assume $\alpha = 2$. A packet transmitted by a node u is successfully received by a node $v \in N_r(u)$ if and only if the received SNR exceeds a threshold β, i.e., $\lambda r^2/d(u, v)^2 > \beta$. Also, $\beta = \frac{1}{2}$ throughout this work. Here, it must be noted that the exact value of β does not alter our inferences, since it merely scales the results. This simple model allows to treat noise as an erasure phenomenon rather than fading, thereby allowing rateless codes to be an excellent choice for data dissemination. A more realistic treatment of an application of rateless codes to fading channels can be found in [23].

A medium access control (MAC) scheme for packet collision avoidance is considered in simulations. Nodes that have some packets to send contend to transmit over the wireless medium in a way that no collision occurs. The MAC enables a fair and realistic comparison of the *latency* of different broadcasting schemes. Here, latency refers to the time taken for all nodes in the network to successfully decode the data sent by the source s. To avoid collisions and the hidden terminal problem, MACs generally require two-hop neighbors of a transmitting node to be silent. This is achieved by incorporating a slotted CSMA scheme with mini-backoff [24]. As detailed in [25], each time unit in this MAC scheme has three parts – a contention period for *request to send* (RTS) transmissions, a period for *clear to send* (CTS) transmissions, and finally a period for actual packet transmission. Though the duration of RTS and CTS messages in comparison to the data transmission period under this MAC affects the actual latency of each algorithm, its effect on each of the broadcast scheme is the same, thereby enabling a fair comparison.

Standard networks such as grid networks and randomly deployed networks are considered in this paper. In grid networks, the source is located in one of the four corners of a square grid of l rows and l columns with neighbors spaced equally away from each other. The transmission range r is chosen to be equal to the distance between any two neighboring nodes constraining the maximum degree of a node in the network to be four. The ideas and results in this case can be directly generalized to square networks of $2l - 1$ rows and columns with the source in the center [26].

In random deployment networks, $n - 1$ non-source nodes are placed independently and uniformly at random in a field of $A \times A$ sq. units, and the source is at $(\frac{A}{2}, \frac{A}{2})$. Asymptotically almost sure connectivity is ensured by assuming that transmission range r_n exceeds r_n^*, the asymptotic threshold of almost sure connectivity radius [27]. Equivalently,

$$\lim_{n\to\infty} \frac{r_n}{r_n^*} > 1, \text{ where } r_n^* = A\sqrt{\frac{\log n}{\pi n}}. \quad (1)$$

IV. FTS: Fractional Transmission Scheme

FTS is based on the idea that various neighbors of a node u can share the load of packet transmission to u. It suffices that each neighbor of a node just sends a fraction of the data such that the total sum of all fractions received by the node from its neighbors is enough for successful data recovery. However, packets from different neighbors must be innovative to ensure that transmissions are not redundant. This is enabled by the use of rateless codes. Unlike the single-hop case, the proposed scheme does not guarantee optimality in terms of energy-efficiency or the total number of transmissions in the network. However, at a node level, optimality in the sense of a small decoding overhead holds. Also, in the proposed scheme, we employ the *decode, re-encode and forward* paradigm. While a forwarding-based scheme is favorable in sparse networks, this paradigm suits densely-deployed networks, since mere forwarding results in redundant receptions and an increase in delay for successful decoding.

FTS uses hop-distance information to partition the neighborhood of each node into two sets. The first, called the *parent set* represents the neighbors of a node from which the node receives packets. The second, called the *children set* that represents the neighboring nodes to which a node must transmit packets after decoding and re-encoding. FTS defines the parent set $\mathcal{P}_r(v)$ of node v as:

$$\mathcal{P}_r(v) = \left\{ w \in N_r(v) : \begin{matrix} \mathbf{H}(w) < \mathbf{H}(v) \text{ or} \\ (\mathbf{H}(w) = \mathbf{H}(v)) \wedge (id(w) < id(v)) \end{matrix} \right\}.$$

In the above definition, w is a parent for v, if it is closer to the source hop-wise. The second clause in the parent set definition eliminates the event that two neighboring nodes depend on each other for successful recovery of source data. Further, FTS defines the children set of a node v as $\mathcal{C}_r(v) = N_r(v) \backslash \mathcal{P}_r(v)$. The details of FTS are as follows.

A. Description of FTS

FTS includes three phases: *Initial Fraction Exchange Phase*, *Fraction Reduction Phase*, and *Data Transmission Phase*. In the first two phases, each node determines the fraction of data that it has to send to its children, and the last phase is the transmission phase. While it is possible to design a scheme where a small subset of nodes do the majority of packet delivery, FTS is designed to practically effect load balancing. The first two phases achieve this goal in a practical fashion by allowing each node to locally ascertain the fraction of data it needs to transmit to its children. As a result, almost all nodes contribute to data delivery.

In *Initial Fraction Exchange Phase* (Algorithm 1), each node v determines the number of neighbors $\kappa_v = |\mathcal{P}_r(v)|$ that will send data to v. Consequently, v expects $\lceil \frac{n_p}{\kappa_v} \rceil$ packets (equivalently, a fraction of $\frac{1}{\kappa_v}$) from each node in $\mathcal{P}_r(v)$. Once v determines this fraction, it declares this fraction to $\mathcal{P}_r(v)$. Each node w collects the list of fractions that it has to send to its children $\mathcal{C}_r(w)$. Each node then sets the maximum entry in its list as the required fraction of data that it has to send. For example, if $\mathcal{C}_r(w) = \{u, v\}$ with $\kappa_u = 3$ and $\kappa_v = 4$, this means that u and v expect one-third and one-fourth of the data from w, respectively. Then, $\alpha_w = \max\left(\frac{1}{3}, \frac{1}{4}\right) = \frac{1}{3}$.

On completing this phase, it is possible that for certain nodes, the sum total of the fractions determined by its parent nodes might be larger than one. In other words, it is possible for a node $u \in V$ to have $\sum_{w \in \mathcal{P}_r(u)} \alpha_w > 1$. Therefore, an attempt to further reduce fractions is attempted by the *Fraction Reduction Phase* (Algorithm 2). In this phase, each node v instructs nodes in $\mathcal{P}_r(v)$ to reduce their fractions equally by $f_v := \frac{1}{\kappa_v} \max\{0, (\sum_{w \in \mathcal{P}_r(v)} \alpha_w) - 1\}$, the amount over and above the required fraction of data that v receives from $\mathcal{P}_r(v)$. Each node v then transmits f_v to its neighbors. A node will reduce its fraction by the minimum of requested reduction in fractions. The new fraction that node w will send is denoted by α'_w. Figure 1 shows a small part of an example network. Suppose $\mathcal{P}_r(u_1) = \{w_1\}$, $\mathcal{P}_r(u_2) = \{w_1, w_2\}$, and $\mathcal{P}_r(u_3) = \{w_2, w_3, w_4\}$, $\mathcal{C}_r(w_1) = \{u_1, u_2\}$, $\mathcal{C}_r(w_2) = \{u_2, u_3\}$, $\mathcal{C}_r(w_3) = \{u_3\}$, and $\mathcal{C}_r(w_4) = \{u_3\}$. We have $\kappa_{u_1} = 1$, $\kappa_{u_2} = 2$, and $\kappa_{u_3} = 3$. Therefore, $\alpha_{w_1} = 1$, $\alpha_{w_2} = \frac{1}{2}$, and $\alpha_{w_3} = \alpha_{w_4} = \frac{1}{3}$. In the fraction reduction phase, we have $f_{u_1} = 0$, $f_{u_2} = \frac{1}{4}$, $f_{u_3} = \frac{1}{18}$. The final fractions will therefore be $\alpha'_{w_1} = 1$, $\alpha'_{w_2} = \frac{1}{2} - \frac{1}{18} = \frac{4}{9}$, and $\alpha'_{w_3} = \alpha'_{w_4} = \frac{1}{3} - \frac{1}{18} = \frac{5}{18}$.

Once the first two phases are completed, we commence the *Data Transmission Phase* (Algorithm 3). First, the source is the only node in the network to be in the *transmit phase*, which is marked by $send\_phase(s) = 1$. In this phase, once a node v receives $\max(\alpha_w - f_v, 0)n_p$ packets from a neighbor w it sends acknowledgement of partial completeness $P_{ack}(v \to w)$, and when it receives encoded packets sufficient to decode, it sends an acknowledgement of completeness $C_{ack}(v)$ to its neighbors. When the latter occurs, the node v decodes the data and commences its send phase, which is marked by $send\_phase(v) = 1$. It then decodes and re-encodes data using a rateless code and sends new encoded packets until all children of v are either complete or do not need any more packets from v. Once the node v has

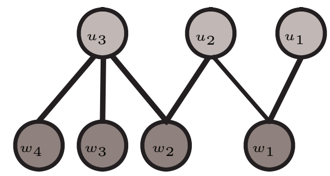

Fig. 1. An illustrative example.

Algorithm 1 *Initial Fraction Exchange Phase*

for $v \in V \setminus \{s\}$ **do**
  $\alpha_v = 0.$
end for
$\alpha_s = 1.$
for $v \in V$ **do**
  transmit: $(v, \mathbf{H}(v), id(v))$
end for
for $v \in V$ **do**
  transmit: $(v, \mathbf{H}(v), \kappa_v = |\mathcal{P}_r(v)|)$
end for
for $v \in V$ **do**
  for $w \in N_r(v) \backslash \mathcal{P}_r(v)$ **do**
    $\alpha_v = \max(\alpha_v, \frac{1}{\kappa_w})$
  end for
  transmit: (v, α_v)
end for

Algorithm 2 *Fraction Reduction Phase*

for $v \in V$ **do**
  transmit: $f_v = \max\left(\frac{1}{\kappa_v} \sum_{w \in \mathcal{P}_r(v)} \alpha_w - \frac{1}{\kappa_v}, 0\right)$
end for
for $v \in V$ **do**
  $\alpha'_v = \max(\alpha_v - \min_{w \in \mathcal{C}_r(v)} f_w, 0)$
end for

Algorithm 3 *Data Transmission Phase*

$send\_phase(s) = 1$
transmit: $n_p\gamma$ encoded packets
for $v \in V \setminus \{s\}$ **do**
  $send\_phase(v) = 0$, $done\_phase(v) = 0$
end for
while $\exists\, v \in V$ with $send\_phase(v) = 0$, $done\_phase(v) = 0$ **do**
  $A = \{u \in V : send\_phase(u) = 0, done\_phase(u) = 0\}$
  for $w \in A$ **do**
    if w has received $n_p\gamma$ encoded packets **then**
      decode n_p data packets
      transmit: $C_{ack}(w)$
      set $send\_phase(w) = 1$
    else
      if w has received $\max(\alpha_z - f_w, 0)n_p\gamma$ packets from neighbor z **then**
        transmit: $P_{ack}(w \to z)$
      end if
    end if
  end for
  $B = \{u \in V : send\_phase(u) = 1\}$
  for $t \in B$ **do**
    if all neighbors have either sent a C_{ack} or a P_{ack} directed at t **then**
      set $send\_phase(t) = 0$, $done\_phase(t) = 1$
    else
      generate an encoded packet P.
      transmit: P
    end if
  end for
end while

heard an acknowledgment (partial or complete) from each of its children, it terminates the transmit phase by setting $done\_phase(v) = 1$ and $send\_phase(v) = 0$. Note that the fractions ascertained in the former two stages are not utilized by the nodes that are in their transmit phases. A node in transmit phase encodes and transmits until it is instructed to terminate encoding by each of its children. As is typical in rateless applications, receiving nodes signal the termination of encoding in transmitting nodes.

Lastly, we would like to highlight the role of P_{ack} in the third phase. Suppose that the third phase is devised without partial acknowledgements i.e., nodes only acknowledge completeness. Consider the example of Fig. 1 and the scenario that w_4, w_3, w_2, and w_1 enter their third phases in that order, and separated by a big gap in time. Then, w_4 will successfully transmit all packets for u_3. The acknowledgement of completeness from u_3 will prevent w_3 from transmitting any packets. However, w_2 will commence its transmit phase later and convey all packets to u_2 before w_1 starts transmitting. Therefore, u_2 will acknowledge before w_1 transmits an encoded packet. However, due to the lack of any acknowledgement from u_1, w_1 will send all packets to u_1. Thus, a total cost of three transmissions/packet will be incurred. Note that in such a format for third phase, the first two phases can become redundant. However, when P_{ack} is present, the total cost is limited to two transmissions/packet. Thus, an improvement in energy cost is obtained by terminating each w_i's encoding appropriately using partial acknowledgements.

B. Discussion of Various Overheads

The proposed scheme has four overhead messages/signals whose effects on latency and transmission cost are described below. Since the network is assumed to be stationary, the setup overheads correspond to the *Initial Fraction Exchange Phase* and the *Fraction Reduction Phase*, and have to be performed just once. In the former phase, the determination of hop-counts and initial fractions can be initiated by the source by flooding a single packet periodically.

The second overhead is the rateless coding overhead γ that corresponds to the ratio of the least number of packets that are required at the decoder to attain a fixed decoding failure probability to the number of input packets n_p. Note that the coding overhead of the maximum likelihood decoder is smaller than that of iterative decoding. However, iterative decoding offers an attractive decoding complexity. In this work, we set $\gamma = 1.03$ for encoding $n_p = 2000$ packets. This choice ensures that the probability of decoding failure under iterative decoding is no more than 10^{-8} [13].

The third overhead is the bits needed to be appended to the header of an encoded packet to identify the indices of packets that were XORed to form the current packet. In rateless codes, an average of $\Theta(\log_2 n_p)$ bits must be appended to each output packet. The overhead imposed by these additional bits is considerably smaller than the corresponding overhead of RLC.

The last overhead is that of acknowledgements in the third phase of FTS. In the presence of losses, while each node transmits C_{ack} is transmitted $\Theta(1)$ times, P_{ack} is transmitted $O(nr_n^2)$ times. A total of $\Theta(n^2 r_n^2)$ partial acknowledgements are sent throughout the network and any reduction in their size results in considerable energy savings.

C. Analysis of FTS in Grid Networks

This section summarizes our results on grid networks. Details of proofs of the results can be found in [25] and [26]. For simplicity, lossless network case is first considered. A simple extension to the case with signal attenuation and Rayleigh fading channels is then presented.

1) Lossless Grid Networks: The following lemma presents the result on the average number of transmissions per packet per node for broadcasting in a grid when the source is at a corner of the grid.

Lemma 1 (Lem. 1 [26]): For an $l \times l$ grid network with the source at the corner of the grid, the average number of transmissions per packet per node under FTS without the second phase and when a rateless code of overhead γ is used is given by $\mathcal{N}_{/p/n} = \frac{(l^2+2l-4)\gamma}{2l^2}$.

Further, when the source is at a corner, the Fraction Reduction Phase alters the fraction of exactly one node – the node that is two hops from the source but not on any face of the grid. This change, however, is asymptotically negligible. Assuming that each node has a transmission range of r, $\mathcal{C}_{/p/n} = \mathcal{N}_{/p/n} r^2$ and from the above lemma, we note that FTS requires $\mathcal{C}_{/p/n} = \frac{\gamma r^2}{2}(1 + o(1))$ asymptotically. To compare the costs of broadcasting using FTS and NC, we fix the direction of edges so that a node with lower hop-count can transmit to its neighbors with higher hop-count. The following result shows that energy cost of FTS and NC are asymptotically close.

Lemma 2 (Lem. 2 [25]): For an $l \times l$ grid network with the source at the corner of the grid and with edge directions such that a node with a smaller hop-distance transmits to its neighbor with larger hop-distance, using NC, we obtain $\mathcal{N}_{/p/n} = \frac{(l^2+l-2)}{2l^2}$.

From the above lemma, we note that, asymptotically, NC makes $\frac{1}{2}$ transmissions per packet per node for broadcasting in the assumed directed graph. It should be noted that the link directions chosen in the above lemma are not optimal, and we can possibly reduce the transmission cost by choosing a different set of directions. However, $\frac{1}{3}$ is a lower bound for the $\mathcal{N}_{/p/n}$ for any broadcasting scheme over an arbitrary (connected) directed grid network. This follows from the fact that each node in a connected directed grid can have at most three outgoing edges and hence each transmission can benefit at most three nodes. Interestingly, for an undirected grid, $\mathcal{N}_{/p/n} = \frac{1}{3}$ can be achieved asymptotically by a simple forwarding/routing scheme [26].

2) Lossy Grid Networks: In this setting, transmissions are subject to distance attenuation and Rayleigh fading as described in Section III. Assuming that the nodes have a transmission range r and two neighboring nodes in a grid are apart by distance r, we have

$$\Pr\{\text{A packet is successfully received}\} = \Pr\{\lambda > \beta\} = e^{-\beta}.$$

Therefore, each node v on the average needs to transmit $\frac{\alpha'_v n_p}{e^{-\beta}}$ packets instead of $\alpha'_v n_p$ packets. We observe that the average costs $\langle \mathcal{N}_{/p/n} \rangle$ and $\langle \mathcal{C}_{/p/n} \rangle$ scale by factor $\frac{1}{e^{-\beta}}$ in comparison to the lossless setting. Since in this work, we have $\beta = \frac{1}{2}$, we notice that for a pair of nodes, the effect of Rayleigh fading for the grid with neighboring nodes placed at a distance exactly equal to the transmission radius is a wireless erasure channel with an erasure probability of $1 - e^{-\frac{1}{2}}$.

D. Analysis of FTS on Lossless Randomly Deployed Networks

In this section, we analyze FTS to derive lower and upper bounds on the expected cost of transmission per packet per node assuming an absence of losses in the wireless medium. Both bounds use an estimation of $\kappa_v = |\mathcal{P}_r(v)|$ for each node v to estimate the expected fraction to be transmitted by each node. Also, as mentioned before, the transmission radius is assumed to be $r_n = \zeta r_n^*$ with $\zeta > 1$, thereby guaranteeing connectivity asymptotically almost surely.

1) Lower Bound on the Performance of FTS: First, we notice that to derive a lower bound for $\mathcal{N}_{/p/n}$, one must bound from above the numbers κ_v for each node v. Each node v in its neighborhood sees Y_v neighbors, where Y_v is a binomial random variable with parameters $n-1$ and $\frac{\pi r_n^2}{A^2}$. Thus, in a given deployment, v can listen to at most Y_v nodes. Thus $\kappa_v \leq Y_v$. However, this evaluation naïvely assumes that every node receives from all its neighbors, which is unlikely in practice. The following result presents a rigorous extension of the above argument.

Theorem 1: $\mathcal{N}_{/p/n}$ for FTS without the second phase over a random deployment network of n nodes with transmission radius $r_n = \zeta r_n^*$ with $\zeta > 1$ satisfies $\frac{n\pi r_n^2 \mathcal{N}_{/p/n}}{A^2} \geq 1$ a.a.s.

Proof: Please refer to Appendix B. ■

2) Upper Bound on the Performance of FTS: To estimate an upper bound on the expected fraction of transmission, one must find a lower bound on the number of parent nodes to which each node will listen. Consider the following scenario as illustrated in Fig. 3, where a node w is situated geographically at point A at a distance $D > r_n$ units away from the source at point S. Suppose that $H(w) = i > 1$. The nodes in the transmission range of w that are closer to the source than w are those that lie both in the circle centered at point A and that centered at s. From simulations we see that with a very high probability there is at least one node in the neighborhood of w with a hop-distance $i-1$ from the source and is closer to the source than w. Figure 2 presents the variation of the probability $\mathscr{P}(n, r_n)$ that a node in an instance of random graph does not have a neighbor that is closer and has smaller hop-distance from the source. The figure presents simulations performed for $n = 500, 1000$, and 1500, $A = 100$, and for varying transmission radii $\frac{r_n}{r_n^*}$. It is noticed that this probability is monotonically decreasing with increasing n and r_n. We conjecture that asymptotically this event occurs for almost all nodes almost surely. A detailed discussion on this conjecture is presented in Appendix C.

Assuming the above conjecture, for the aforementioned node w located at point A, we see that there exists a node $v \in N_{r_n}(w)$ that is closer to s than w and has a strictly smaller hop-distance. Then, all nodes that are located in the transmission range of both w and v and are closer to the source must have a hop-distance of no more than i. Thus, nodes in

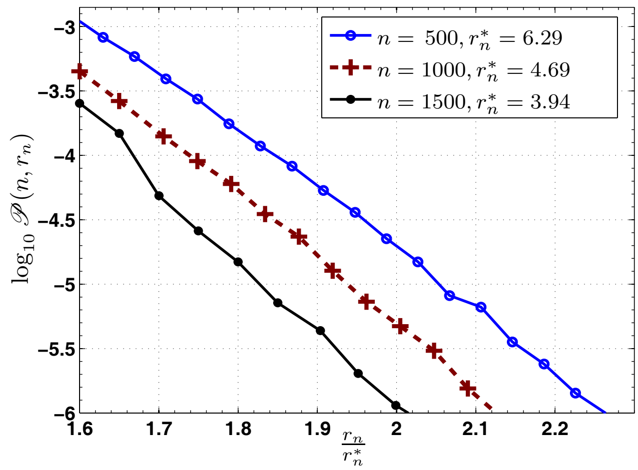

Fig. 2. Variation of $\mathscr{P}(n, r_n)$ with $\frac{r_n}{r_n*}$.

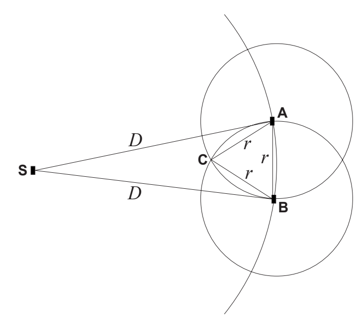

Fig. 3. Illustration of $\Delta(r, D)$ used for deriving the upper bound.

the area common to all the three circles (the circle centered at source passing through v and the circles of radius r_n centered at v and w, resp.) are expected to transmit to w under FTS. It can be shown that the area common to all the circles is minimum when v is located at B – one of the two points of intersection between the two circles centered at A and S as illustrated in Fig. 3. Evaluation of this minimum area yields

$$\Delta(r_n, D) := \frac{4\pi - 3\sqrt{3}}{12} r_n^2 + D^2 \sin^{-1}(\frac{r_n}{2D}) - \frac{r_n}{2}\sqrt{D^2 - \frac{r_n^2}{4}}$$
$$\geq (\frac{\pi}{3} - \frac{\sqrt{3}}{4}) r_n^2 =: \underline{\Delta}(r_n). \tag{2}$$

By the design of FTS, every node in a region of area $\underline{\Delta}(r_n)$ transmits to w, since every node in this region has a hop-count of at most i and is closer than w. Therefore, for every $w \in V$ such that $d(w, s) > r_n$, there is a region A_w of area $\underline{\Delta}(r_n)$ such that all nodes in A_w are parents of w under FTS. Without affecting the results, we can drop the distance clause $d(w, s) > r_n$, since Thm. 2 guarantees that there are at most $\frac{\pi r_n^2 G_>^{-1}(\zeta^{-2})}{A^2} = o(n)$ nodes within a distance of r_n from s. Let X_w be the random variable denoting number of nodes in the region A_w (other than v). Since this region A_w is not strictly a function of the position of w alone, the distribution X_w is not clear. However, a reasonable assumption to make is that X_w is binomial with parameters $n-2$ and $q_n = \frac{\underline{\Delta}(r_n)}{A^2}$. Also note that X_w is a local random variable depending on the events that occur within a radius r_n around w. Using the above assumptions and notations, we have

$$\alpha_v = \max_{w\in\mathcal{C}_v} \frac{1}{|\mathcal{P}_w|} \le \max_{w\in\mathcal{C}_v} \frac{1}{1+X_w} \le \max_{w\in V} \frac{\mathbb{I}_{r_n}(d(v,w))}{1+X_w} =: \tau_v.$$

Define $\mathcal{Z} := \sum_{v\neq s} \tau_v$. Then $\mathcal{N}_{/p/n} \le \frac{1}{n}(1+\mathcal{Z})$. Also note that $\{\tau_v : v \neq s\}$ are identically distributed. Fix $v, v' \in V\backslash\{s\}$ with $v \neq v'$. Then, $E[\mathcal{Z}] = (n-1)E[\tau_v]$. To compute $E[\tau_v]$, just as in Appendix B, we set $W = |N_{2r_n}(v)|$ and $p_n := \frac{\pi r_n^2}{A^2}$. Then $\mu_W = E[W] = \frac{4\pi(n-1)r_n^2}{A^2}$, and for $\delta \in (0,1)$,

$$\Pr[|\frac{W}{\mu_W} - 1| \le \delta] \overset{(8,9)}{\le} e^{-\mu_W G_>(1+\delta)} + e^{-\mu_W G_<(1-\delta)}. \quad (3)$$

Further, conditioned on $\mathcal{Y} = \{|N_{2r_n}(v)| = m+1\}$, the distribution of X_w for a node $w \in N_{r_n}(v)$ is given by

$$\Pr[X_w = j|\mathcal{Y}] = \binom{m}{j} \frac{q_n^j}{(2p_n)^j}(1 - \frac{q_n}{2p_n})^{m-j}$$

$$\therefore \Pr[X_w \le (1-\delta)^2 nq_n|\mathcal{Y}] \le e^{-G_<\left(\frac{4(1-\delta)^2 np_n}{m}\right)\frac{mq_n}{4p_n}}.$$

Let $\mathcal{E} := \{|\frac{W}{\mu_W} - 1| \le \delta\}$ and $\varrho := \Pr[\tau_v > \frac{1}{(1-\delta)^2 nq_n}]$. Then,

$$\begin{aligned} \varrho &\le \Pr[\tau_v > \frac{1}{(1-\delta)^2 nq_n}|\mathcal{E}] + \Pr[\mathcal{E}^c] \quad (4)\\ &\le \Pr[\exists w \in N_{r_n}(v) \text{ s.t. } X_w < (1-\delta)nq_n|\mathcal{E}] + \Pr[\mathcal{E}^c]\\ &\overset{(a)}{\le} (1+\delta)\mu_W e^{-(1-\delta)^2 nq_n G_<(1-\delta)} + \Pr[\mathcal{E}^c] \to 0, \end{aligned}$$

where (a) follows from union bound and (3). Thus, $\forall\, \delta > 0$, $\Pr[\tau_v \le \frac{1}{(1-\delta)^2 nq_n}] \to 1$. Therefore, $nq_n E[\tau_v] \ge 1$ asymptotically. Further,

$$E(\mathcal{Z}^2) = (n-1)E[\tau_v^2] + 2\binom{n-1}{2}E[\tau_v\tau_{v'}], \text{ and}$$

$$\begin{aligned} E[\tau_v\tau_{v'}] &\overset{(b)}{\le} E_{>4r_n}[\tau_v\tau_{v'}] + \frac{16\pi r_n^2}{A^2}E_{\le 4r_n}[\tau_v\tau_{v'}]\\ &\overset{(c)}{=} E^2[\tau_v] + \frac{8\pi r_n^2}{A^2}E_{\le 4r_n}[\tau_v^2 + \tau_{v'}^2]\\ &\overset{(d)}{\le} E^2[\tau_v] + \frac{16\pi r_n^2}{A^2}\Theta(\frac{1}{\log^2 n}) \end{aligned}$$

$$\begin{aligned} \therefore E[\mathcal{Z}^2] &\le (n-1)^2E^2[\tau_v] + (n-1)\mathrm{var}(\tau_v) + o(n^2E[\tau_v^2])\\ &= E^2[\mathcal{Z}] + o(E^2[\mathcal{Z}]). \quad (5) \end{aligned}$$

Note that in the above, (b) follows by splitting the expectation operator by conditioning on the event that $d(v,v') \le 4r_n$, and $E_{>4r_n}$ and $E_{\le 4r_n}$ denote the respective conditional expectation operators; (c) follows from the fact that when $d(v,v') > 4r_n$, the random variables τ_v and $\tau_{v'}$ are asymptotically uncorrelated due to their local nature; and (d) follows from the assumption that $\tau_v, \tau_{v'}$ are binomial, and their maximum is asymptotically of the same order as that of the average degree. Application of Chebychev inequality confirms the concentration of $\mathcal{Z}$ around its mean. Therefore,

$$\mathcal{N}_{/p/n} = \frac{1}{n} + \frac{1}{n}\sum_{v\neq s}\alpha_v \le \frac{1+\mathcal{Z}}{n} \overset{\text{a.a.s.}}{\le^*} \frac{A^2}{n\underline{\Delta}(r_n)}(1+o(1)).$$

Note that when the radius of connectivity is $r_n = \zeta r_n^*$ with $\zeta > 1$, the above result and Thm. 1 together imply

$$1 \le \left(\frac{\pi n r_n^2}{A^2}\mathcal{N}_{/p/n}\right) \le^* \frac{12\pi}{(4\pi - 3\sqrt{3})} \quad \text{a.a.s.} \quad (6)$$

thereby establishing the asymptotic order of variation of $\mathcal{N}_{/p/n}$. Note that the upper bound is not based on a proof, but on an argument based on reasonable assumptions motivated from simulations. This fact is highlighted by the notation $\le^*$ in above equations. To compare the cost of FTS with the optimal scheme, notice that the size of an MCDS is at least $\frac{n}{\overline{M}_n}$, where $\overline{M}_n$ is the maximum degree of a node in the random deployment. From Thm. 2, we notice that the minimum asymptotic cost of broadcasting $\mathcal{N}_{/p/n,\min}$ is at least

$$np_n\mathcal{N}_{/p/n,NC} \ge np_n\mathcal{N}_{/p/n,\min} \ge \frac{np_n}{\overline{M}_n} \overset{\text{a.a.s}}{\ge} \frac{1}{G_>^{-1}(\zeta^{-2})}. \quad (7)$$

Thus, we see that we can achieve the same order of broadcast cost per packet per node asymptotically when employing FTS. Finally, in the presence of noise phenomena, the transmission cost per packet per node for random deployment networks gets appropriately scaled just as in the case of grid networks.

V. Simulation Results

To compare the energy efficiency of FTS with other broadcasting algorithms such as MPR, DP, BIP, and NC, simulations were performed on both square grids and randomly deployed networks. Though all the schemes aim at constructing a dominating set, DP and MPR are localized algorithms, whereas BIP is centralized. Owing to their random/greedy nature, these algorithms are hard to analyze over the random deployment setting and have been studied using simulations.

For simulations, the rateless code overhead γ was set at 1.03, since this choice is sufficient to guarantee a success probability of over $1 - 10^{-8}$ when transmitting $n_p \ge 2000$ packets [13]. The channel model and MAC scheme as explained in Section III were considered for these simulations. The size of the packets was assumed to be the same in all schemes. Also, it is worth noting that the amount of information payload per packet is higher for FTS in comparison with NC as described in Section IV-B.

Schemes such as BIP and MPR are originally tailored for lossless networks, and they do not guarantee reliability in lossy networks. However, they can be extended to lossy networks using either multiple retransmissions or forward error correction at each link. In this way, if a channel has a corresponding loss probability of ϵ, the number of transmissions will be scaled by a factor of $\frac{1}{1-\epsilon}$ on the average.

A. Grid Networks

Simulations were performed for varying grid sizes to evaluate the transmission costs and latency of various schemes. Figure 4 depicts the transmission cost $\mathcal{C}_{/p/n}$ for different reliable broadcasting schemes for grid networks of varying sizes with source at the corner. It is assumed that any two adjacent nodes is the same as transmission radius, i.e., $r = 1$. The transmissions are subject to distance attenuation and Rayleigh fading with probability of successful delivery of a packet as $e^{-\frac{1}{2}}$, since $\beta = \frac{1}{2}$. For FTS, both simulation and analytical results as derived in Section IV-C are presented. As we can see, the simulation and analytical results match. We also depict the cost of broadcasting with NC. For large grid networks, the only difference in the cost of broadcasting

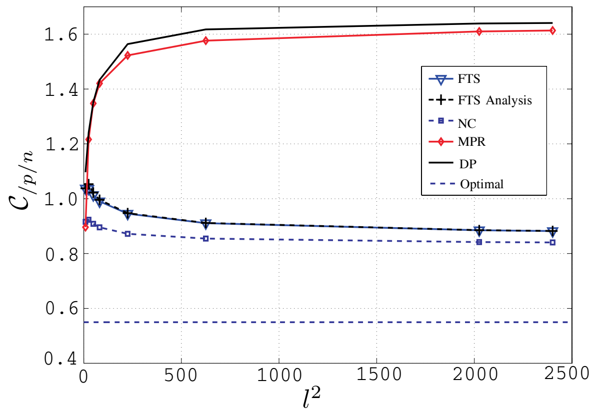

Fig. 4. $\mathcal{C}_{/p/n}$ vs grid size for broadcasting schemes over lossy grids.

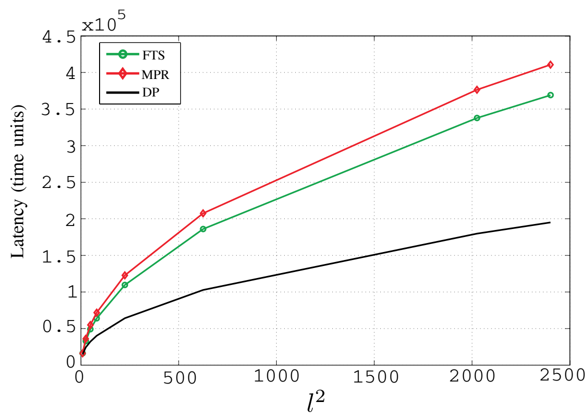

Fig. 5. Latency of schemes in lossy $l \times l$ grids with the source at a corner.

between FTS and NC is the factor γ, the overhead of rateless coding. For NC, it is assumed that RLC over a large field $GF(q)$ results in no coding overhead (for the price of higher complexity of decoding). As the number of packets n_p is increased, γ approaches one, and FTS and NC result in the same energy consumption. The asymptotic optimal cost of $\frac{1}{3e^{-\frac{1}{2}}}$ obtainable from the optimal scheme detailed in [26] is also presented in Fig. 4. Note that NC does not offer the asymptotically-optimal cost here because of the sub-optimality of the assumed edge directions.

Figure 5 depicts the latency for various schemes for reliable delivery of 2000 packets in lossy $l \times l$ grids with the source at the corner. The latency of NC is not included, since it is unclear how to integrate the MAC layer and interference in NC. Nevertheless, we expect NC to have a lower latency than FTS, since the latter uses a decode, re-encode and forward paradigm. Here, it must be noted that the energy consumed by all schemes remain unaltered when the ideal collision-free MAC (with two-hop silence) is used instead of the one chosen in this work. However, the latencies of the various schemes will differ based on the chosen MAC.

B. Random Deployment Network

Here, we consider $n - 1$ non-source nodes each with transmission range r randomly deployed in an area 100×100 square units, for different values of n and r. The source is assumed to be at the center of the area. The transmission range r in each case is selected such that the resulting graph is connected. Transmissions are subject to attenuation due to distance and Rayleigh fading with parameter $\beta = \frac{1}{2}$. First, we compare FTS and NC. To compare FTS with NC, the assignment of directions for NC is set to be the same as that for FTS. Note that the edge directions for FTS are based on hop-distances, as is described in Section IV. Table I compares $\mathcal{C}_{/p/n}$ for FTS and NC for varying n and r in a region of 100×100 sq. units. Clearly, given the same directions as FTS, NC has a lower cost, since it involves flow optimization for the selected directions. However, the excess cost of FTS is less than 8% in comparison with that of NC. Thus, FTS can be seen to be a simple, easily-implementable broadcasting option for such networks with an energy cost relatively close to that of NC.

TABLE I
$\mathcal{C}_{/p/n}$ IN RANDOM NETWORKS

n	r	Approach	$\mathcal{C}_{/p/n}$
15	23	NC	331.10
		FTS	357.64
20	22	NC	326.04
		FTS	336.61
25	21	NC	300.29
		FTS	323.05

Next, we consider networks of up to 500 nodes. Due to the complexity of the optimization sub-routine, we were unable to simulate NC for this setting. Therefore, we compare FTS with DP, MPR, and BIP for large networks.

In BIP, nodes can have different transmission ranges to decrease the cost of broadcasting. Given the option that nodes can change their transmission range, we can extend FTS to FTS$_{adapt}$. FTS$_{adapt}$ is similar to FTS except that every node v in the network has the option of reducing its transmission range from r_n to r_{nv}, where r_{nv} is the distance between v and the farthest child of v that is going to listen to v in the data transmission phase (Algorithm 3). Thus, the probability that a child w of v receives a packet from v decreases, since we have

$$\Pr\{w \text{ receives a packet from } v\} = \Pr\{\lambda > \beta d^2(v,w)/r_{nv}^2\} = e^{-\beta d^2(v,w)/r_{nv}^2} \le e^{-\beta d^2(v,w)/r_n^2}.$$

Therefore, in FTS$_{adapt}$ more transmissions are needed. However, each transmission has less cost. Overall, we can expect FTS$_{adapt}$ to be more energy-efficient than FTS.

Figure 6 compares $\mathcal{C}_{/p/n}$ for FTS, FTS$_{adapt}$, MPR, DP, and BIP. BIP offers the best performance, whereas the two proposed schemes are the next best choice. However, we should note that BIP is a centralized scheme that needs global knowledge of the network, and is computationally intensive. In contrast, FTS and FTS$_{adapt}$ are distributed schemes with low complexity. Comparing the performance of FTS$_{adapt}$ and FTS with distributed schemes DP and MPR, we see that FTS and FTS$_{adapt}$ have much better performance. For large n, the improvement due to adaptive transmission range is small because of the fact that there are $\Theta(\log n)$ nodes uniformly distributed in the neighborhood of each node. Consequently, the distance between each node and its farthest neighbor is close to r_n.

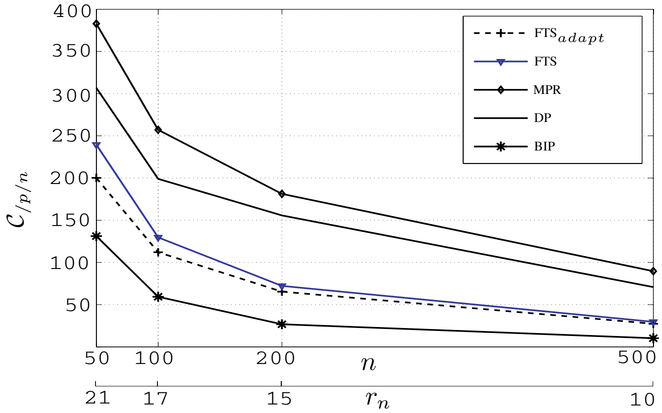

Fig. 6. $\mathcal{C}_{/p/n}$ of different broadcasting schemes in random networks.

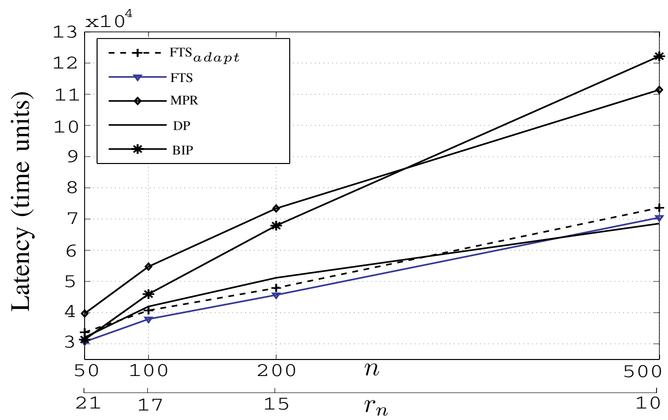

Fig. 7. Latency of different schemes for broadcasting 2000 packets over a random network.

Figure 7 depicts the latency of different broadcasting schemes. DP seems to have the best latency whereas FTS and FTS$_{adapt}$ guarantee slightly worse latency performance. FTS$_{adapt}$ has slightly larger latency than FTS, which can be attributed to the increase in the number of transmission due to reduced transmission power in FTS$_{adapt}$.

In order to compare our analytical bounds derived in Section IV-D with the actual performance of FTS, we simulated FTS (without second phase) for networks of various sizes assuming no channel losses. Figure 8 presents the upper bound of (6) and the lower bound of Theorem 1 in comparison to the simulation results for FTS. It also presents the lower bound for the minimum possible cost of transmission per packet per node, which is based on the maximum degree of nodes in the network. It can be seen that the upper bound is fairly accurate in predicting the actual performance of FTS for large networks.

VI. Conclusion

In this work, the application of rateless codes for broadcasting in multihop wireless networks is explored. A reliable and scalable broadcasting algorithm known as the fractional transmission scheme is proposed. The scheme exploits local network information and employs rateless coding at each node to reduce unwanted redundancy in transmissions. The cost of transmission of the proposed scheme is derived for both grid and random deployment networks and a comparison of our

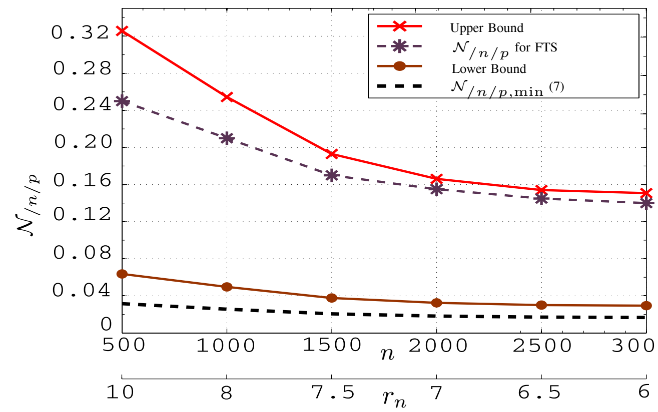

Fig. 8. Illustration of the analytical bounds derived for FTS.

scheme with other broadcasting algorithms such as MPR, DP, BIP and NC is presented. Simulation results show that the performance of proposed scheme is comparable to complex schemes such as BIP and NC. Due to its low complexity and competitive performance, FTS is seen to be a practical broadcasting scheme in large-scale wireless ad-hoc networks.

Appendix A

Lemma 3: Let $G_> : [1,e] \to [0,1], G_>(x) = 1 - x + x\log x$, $G_< : [0,1] \to [0,1], G_<(x) = 1 - x + x\log x$, and $G_>^{-1}$, $G_<^{-1}$ be their inverses. Let X be a binomial random variable with parameters n and p. Then, for $\delta \in (0,1)$,

$$P[X > (1+\delta)np] \leq e^{-npG_>(1+\delta)} \qquad (8)$$

$$P[X < (1-\delta)np] \leq e^{-npG_<(1-\delta)} \qquad (9)$$

Proof: This is a restatement of Chernoff bound for binomial RVs [27]. ■

Theorem 2: The max. degree $\overline{M}_n$ and min. degree $\underline{m}_n$ in a random geometric graph of n nodes with transmission radius $r_n = \zeta r_n^*$ with $\zeta > 1$ over a region of $A \times A$ sq. units satisfy

$$G_<^{-1}\left(\frac{1}{\zeta^2}\right) \leq \lim_{n\to\infty} \frac{\underline{m}_n}{\frac{n\pi r_n^2}{A^2}} \leq \lim_{n\to\infty} \frac{\overline{M}_n}{\frac{n\pi r_n^2}{A^2}} \leq G_>^{-1}\left(\frac{1}{\zeta^2}\right) \text{ a.s.} \qquad (10)$$

Proof: The proofs of the exact limits can be found in Thm. 6.14 and Thm. 7.14 of [27]. ■

Appendix B

We note that the fraction that a node v determines at the end of the first stage is bounded below as

$$\alpha_v \geq \min_{w:d(v,w)\leq r_n} \frac{1}{|N_r(w)|} = \overset{*}{\min_{w\in V}} \frac{\mathbb{I}_{r_n}(d(v,w))}{\sum_{u\in V}\mathbb{I}_{r_n}(d(u,w))} =: \beta_v,$$

where $\overset{*}{\min}$ denotes the operator selecting the smallest positive number from a set (and zero if the set is empty). Let $\mathcal{Z} := \sum_{v\neq s}\beta_v$. Since each random variable β_v has the same distribution, we have $E[\mathcal{Z}] = (n-1)E[\beta_t]\ \forall t \in V \setminus \{s\}$. Fix $t \in V \setminus \{s\}$. To compute $E[\beta_t]$, set $W = |N_{2r_n}(t)|$ and $p_n := \frac{\pi r_n^2}{A^2}$, then $\mu_W = E[W] = \frac{4\pi(n-1)r_n^2}{A^2}$, and for $\delta \in (0,1)$,

$$\Pr\left[\left|\frac{W}{\mu_W} - 1\right| \leq \delta\right] \overset{(8,9)}{\leq} e^{-\mu_W G_>(1+\delta)} + e^{-\mu_W G_<(1-\delta)}. \qquad (11)$$

Further, conditioned on $\mathcal{Y} = \{|N_{2r_n}(t)| = m + 1\}$, the distribution of the number of neighbors of a node $w \in N_{r_n}(t)$ is given by

$$\Pr[\partial_w - 1 = j|\mathcal{Y}] = \binom{m}{j}\frac{3^{m-j}}{4^m}$$

$$\therefore \Pr[\partial_w - 1 \geq (1+\delta)^2 np_n|\mathcal{Y}] \leq e^{-G_>\left(\frac{4(1+\delta)^2 np_n}{m}\right)\frac{m}{4}}.$$

Let $\mathcal{E} := \{|\frac{W}{\mu_W} - 1| \leq \delta\}$ and $\rho := \Pr[\beta_t < \frac{1}{(1+\delta)^2 np_n}]$. Then,

$$\begin{aligned}\rho &\leq \Pr[\beta_t < \frac{1}{(1+\delta)^2 np_n}|\mathcal{E}] + \Pr[\mathcal{E}^c] \qquad (12)\\ &\leq \Pr[\exists w \in N_{r_n}(t) \text{ s.t. } \partial_w > (1+\delta)^2 np_n|\mathcal{E}] + \Pr[\mathcal{E}^c]\\ &\overset{(a)}{\leq} (1+\delta)\mu_W e^{-(1-\delta)\mu_W G_>\left(\frac{4(1+\delta)^2 np_n}{(1+\delta)\mu_W}\right)} + \Pr[\mathcal{E}^c] \to 0,\end{aligned}$$

where (a) follows from union bound and (11). Thus, $\forall\,\delta > 0$, $\Pr[\beta_t \geq \frac{1}{(1+\delta)^2 np_n}] \to 1$. Hence, $np_n E[\beta_t] \geq 1$ asymptotically. Now, fix $t \in V \setminus \{s\}$ and $t' \in V \setminus \{s, t\}$. Then,

$$E(\mathcal{Z}^2) = (n-1)E[\beta_t^2] + 2\binom{n-1}{2}E[\beta_t\beta_{t'}], \text{ and} \qquad (13)$$

$$\begin{aligned}E[\beta_t\beta_{t'}] &\overset{(b)}{\leq} E_{>4r_n}[\beta_t\beta_{t'}] + \frac{16\pi r_n^2}{A^2}E_{\leq 4r_n}[\beta_t\beta_{t'}]\\ &\overset{(c)}{\leq} E^2[\beta_t] + \frac{8\pi r_n^2}{A^2}E_{\leq 4r_n}[\beta_t^2 + \beta_{t'}^2]\\ &\overset{(d)}{\leq} E^2[\beta_t] + \frac{16\pi r_n^2}{A^2}\Theta(\frac{1}{\log^2 n})\\ \therefore E[\mathcal{Z}^2] &\leq (n-1)^2E^2[\beta_t] + (n-1)\text{var}(\beta_t) + o(n^2E^2[\beta_t])\\ &= E^2[\mathcal{Z}] + o(E^2[\mathcal{Z}]). \qquad (14)\end{aligned}$$

In the above, (b) follows by conditioning the expectation operator based on the event that $d(t, t') \leq 4r_n$, and $E_{>4r_n}$ and $E_{\leq 4r_n}$ denote the respective conditional expectation operators; (c) follows from the fact that when $d(t, t') > 4r_n$, β_t and β_t' are asymptotically uncorrelated due to their local nature; and (d) holds because the largest value $\beta_t, \beta_{t'}$ can take is the reciprocal of the minimum degree of the graph, which is at least $G_<^{-1}(\zeta^{-2})\zeta^2 \log n$ a.a.s. Therefore, by application of Chebychev inequality, we confirm that $\mathcal{Z}$ is concentrated around its mean. The claim then follows by noticing that

$$\mathcal{N}_{/p/n} = \frac{1}{n} + \frac{1}{n}\sum_{v \neq s}\alpha_v \geq \frac{1+\mathcal{Z}}{n} \overset{\text{a.a.s.}}{\geq} \frac{A^2}{n\pi r_n^2}(1 + o(1)).$$

Appendix C
A Discussion on the Conjecture of Section IV-D

Several approaches in the literature of random geometric graphs have analyzed the relation between Euclidean and hop-distances [28]–[30]. The most relevant result is that of Ellis *et al.* In [30], they showed that when $r_n > r_n^*$, it is asymptotically almost sure that the hop-distance $d_G(s, v)$ between source s and a node v is given by

$$d_G(s, v) \leq \frac{d(v, s)}{r_n}\Big(1 + O\big(\sqrt{\frac{\log\log n}{\log n}}\big)\Big), \qquad (15)$$

where $d(v, s)$ denotes the Euclidean distance between v and s. Notice that the first term of the above equation corresponds to the minimum number of hops taken by a hypothetical straight line path from s to v. The second term provides freedom for paths to wiggle around this straight line due to the randomness in node locations. Figure 9 illustrates the construction of a shortest-hop path for a node v. To use the least number of hops, it would be ideal if nodes are placed regularly r_n apart from each other on the dashed line connecting s and v. However, due to the random realization, we can only identify a path close to this line.

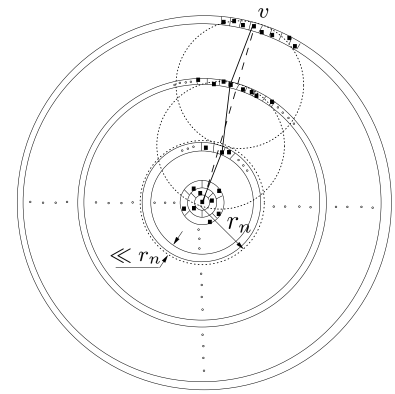

Fig. 9. Existence of short geographic paths.

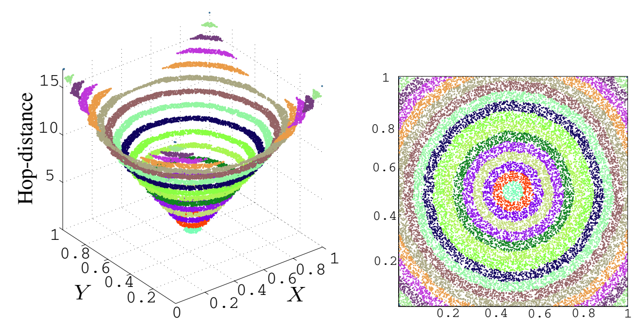

Fig. 10. Illustration of occurrence of annuli in a network with $n = 25000$ and $r_n = 4r_n^*$.

Since the deployment is uniform, the hop-distance of a node can be expected to depend only on the distance from the source. Therefore, nodes with the same hop-distance from the source can be expected to partition the region into annuli. Though this intuition is motivated by ensemble averages, partitioning is also noticed in network realization instances. For example, Fig. 10 depicts the hop-distances of nodes in a network with $n = 25000$, $r_n = 4r_n^*$, and $A = 1$.

The difficulty in quantifying the monotonicity property is pictorially illustrated in Fig. 11. Consider the situation highlighted in Fig. 11(b), where the region of domain is restricted to points jr_n away from s. Suppose that the nodes that are within a distance of $(j-1)r_n$ from s satisfy the monotonicity conjecture. Then, their next-hop neighbors must lie within a union of circles given by

$$\mathcal{B}_{j-1} := \bigcup_{v: d(s,v) \leq (j-1)r_n} B(v, r_n) \qquad (16)$$

Note that this region $B(s, jr_n) \setminus \mathcal{B}_{j-1}$ consists of a thin annulus in addition to "kinks" in the region that are similar to the dark region between points A, B and C of Fig. 11(a). It

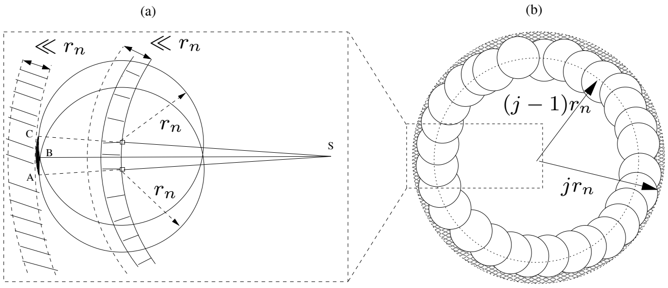

Fig. 11. Illustration of the monotonicity property.

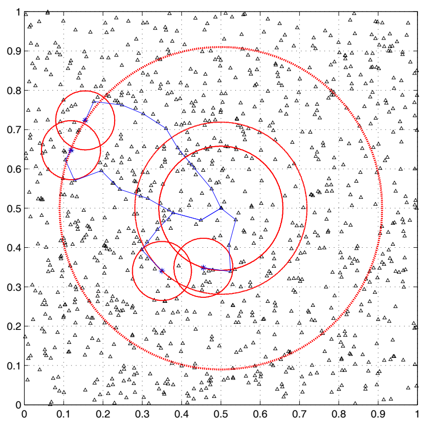

Fig. 12. Examples of non-monotonic paths.

is the presence of these kinks that make the region $\mathcal{B}_{j-1}$ non-convex. While the nodes that fall in $\mathcal{B}_{j-1}$ can be connected by an edge to a node that is in $B(s,(j-1)r_n)$, a node in the kink ABC cannot be connected. In fact, it is possible that a node in such a kink must rely on a node farther away from s to receive forwarded data. However, an analytic quantification of the rarity of such events seems elusive. We conclude the discussion with an illustration of these rare events.

Fig. 12 presents a realization of 1000 nodes with $r_n = 1.6r_n^*$ dispersed in a square area. Only four nodes are non-monotonic, i.e., all shortest-hop paths to these nodes approach them from nodes farther away from the source. The figure presents a shortest-hop path connecting each non-monotonic node to the source. For this setup, each node sees on the average $\frac{n}{2}\pi r_n^2 \sim 8$ nodes in the half-neighborhood that is closer to the source. However, notice that the corresponding half-neighborhood of each non-monotonic node is scarcely populated. Such low density in the neighborhood seems to be the primary reason why shortest-hop paths wind around these node to connect to them. However, in large deployments with appropriately chosen communication radii, Thm. 2 guarantees a healthy occupancy of $\Theta(\log n)$ nodes in the neighborhood of each node, thereby suggesting that these events are an aberration from the norm.

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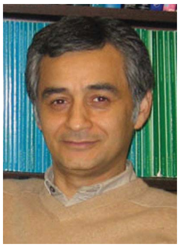

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